Vol. 67, No. 2 DUKE MATHEMATICAL JOURNAL © August 1992

THE CODIMENSION-TWO HOMOLOGY OF THE MODULI SPACE OF STABLE CURVES IS ALGEBRAIC

DAN EDIDIN

Introduction. Let $\overline{\mathcal{M}}_g$ be the compact moduli space of stable curves of genus g. The purpose of this paper is to give an explicit algebraic basis for the (complex) codimension-two rational homology group of $\overline{\mathcal{M}}_g$. This basis makes it possible to explore the enumerative geometry of codimension-two subvarieties of $\overline{\mathcal{M}}_g$ (see for example [Ed]).

The enumerative geometry of codimension-one cycles in $\overline{\mathcal{M}}_g$ has been studied for almost ten years, beginning with work of Diaz and Harris-Mumford, who computed the equivalence classes of certain geometrically defined codimension-one subvarieties of $\overline{\mathcal{M}}_g$ in terms of a set of $[g/2] + 2$ independent elements in the rational Picard group $Pic_{\mathbb{Q}}(\overline{\mathcal{M}}_g)$ ([Di] and [HM]). (The usefulness of these computations is realized in [HM], where it is proved that $\mathcal{M}_g$ is not unirational for g odd and at least 23. Specifically, by computing the class of a natural codimension-one subvariety of $\overline{\mathcal{M}}_g$, the authors showed that the linear series given by a positive multiple of the canonical divisor on $\overline{\mathcal{M}}_g$ contained an effective divisor.)

Both Diaz and Harris-Mumford computed the classes of codimension-one subvarieties whose restriction to $\mathcal{M}_g$ were determinantal varieties associated to naturally occurring vector bundles on $\mathcal{M}_g$. Using Porteous's formula, they were able to show that the restriction of their cycles could be expressed in terms of one class (λ, the first Chern class of the Hodge bundle). Since the boundary $\overline{\mathcal{M}}_g - \mathcal{M}_g$ was known to be the union of $[g/2] + 1$ irreducible subvarieties of codimension-one ([DM]), they could express the classes of these subvarieties in terms of $[g/2] + 2$ classes, one coming from $\mathcal{M}_g$ and the others from the boundary. At the time, it was not known whether these $[g/2] + 2$ classes generated either the homology or Chow groups, or any other interesting group of codimension-one cycles on $\overline{\mathcal{M}}_g$.

In [Ha], Harer succeeded in computing the first two homology groups of $\mathcal{M}_g$. Specifically, he showed that if $g \geqslant 3$, then $H^1(\mathcal{M}_g, \mathbb{Z}) = 0$, and $H^2(\mathcal{M}_g) = \mathbb{Z}$. Using the exponential sequence, it follows that $Pic(\mathcal{M}_g)$ has rank one. Consequently, the $[g/2] + 2$ classes appearing in the computations of Diaz and Harris-Mumford form a basis for $Pic_{\mathbb{Q}}(\overline{\mathcal{M}}_g)$.

Until recently, essentially nothing was known about the group of codimension-two cycles in $\mathcal{M}_g$, or even the codimension-one cycles in $\overline{\mathcal{M}}_g - \mathcal{M}_g$ (since the boundary has codimension one, these are codimension-two cycles in $\overline{\mathcal{M}}_g$). As a result, one could not compute the classes of any codimension-two subvarieties of

Received 23 July 1991. Revision received 9 January 1992.