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L^2-INDEX FOR CERTAIN DIRAC-SCHRÖDINGER OPERATORS

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1. Introduction. In [B2] a formula is given for evaluating the L^2-index of a Dirac-type operator D on a certain class of (noncompact) complete Riemannian manifolds. Although in principle computable, especially in the Fredholm case, this formula contains terms reflecting the contribution of the small eigenvalues, which are difficult to evaluate. We show in this paper that the addition of a skew-adjoint potential V, satisfying reasonable assumptions at infinity, has the effect of eventually overcoming the influence of the small eigenvalues of D. Thus, the L^2-index of the "Dirac-Schrödinger operator" $D + \lambda V$, for λ sufficiently large, is given by an "adiabatic limit" of η-invariants and is therefore local at infinity. (See Theorem 3.2 below.) This generalizes and at the same time explains index formulae of Callias type. (See [C], [A].)

Due in part to the nature of the problem, but mainly because of the limitations of the method we employ, the manifolds we are considering are subject to a number of constraints at infinity. Some of these conditions have a clear geometric meaning, but others do not. Thus, the class of manifolds to which our results apply is not easy to quantify. It seems possible to enlarge it to encompass all complete manifolds of strictly negative sectional curvature and finite volume; Theorem 3.5 below constitutes an important step in this direction.

The L^2-index theorem we prove in this paper can be used in conjunction with vanishing type arguments, much in the same way as the standard index theorem for Dirac operators and its relative version are employed in [GL], to gain information about the scalar curvature. To illustrate this, we discuss in Section 4 a "version with boundary" of the "conservation principle" for the scalar curvature of perturbations of the standard metric on the n-sphere, suggested by Gromov [G] and proved in [L]. We wish to thank Maung Min-Oo for making us aware of these references and for substantial help with the calculations in Section 4.

2. An abstract index theorem. In this section we recall the main facts from [B2], adapted to the following situation. Let M be a complete noncompact Riemannian manifold, of odd dimension $m = 2k + 1$, and let D be a generalized Dirac operator acting on the smooth compactly supported sections of a Clifford bundle E, equipped

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