Vol. 66, No. 2 DUKE MATHEMATICAL JOURNAL © May 1992

WEIGHTED ESTIMATES FOR $\bar{\partial}$ IN DOMAINS IN $\mathbb{C}$

BO BERNDTSSON

1. Introduction. Let D be a domain in $\mathbb{C}$ and let f be a measurable function in D. Assume ϕ is a subharmonic function in D. We want to solve the equation

$$\frac{\partial u}{\partial \bar{z}} = f \tag{1.1}$$

and find estimates for the solution in the L^p-norm with respect to the weight function $e^{-\phi}$. It follows from Hörmander's theorem [H2] that there is a solution to (1.1) such that

$$\left(\int_D |u|^2 e^{-\phi}\right)^{1/2} \leq \operatorname{diam} D \left(\int |f|^2 e^{-\phi}\right)^{1/2} \tag{1.2}$$

provided the right-hand side is finite. Indeed, a similar result holds in any number of variables. It seems that rather little is known concerning the same problem in other norms. In a recent paper by Fornaess and Sibony [FS] the following result is proved.

THEOREM 1. *Let* $1 < p \leq 2$ *and normalize so that* diam $D \leq 1$. *With the assumptions above, there is a solution* u *to* (1.1) *such that*

$$\left(\int_D |u|^p e^{-\phi}\right)^{1/p} \leq C/(p-1)\left(\int_D |f|^p e^{-\phi}\right)^{1/p}$$

provided the right-hand side is finite. Here, C *is an absolute constant.*

Fornaess and Sibony also give examples that show that this estimate is false for $2 < p$, and they ask whether L^p-estimates still hold for $p = 1$. Our first result is that the answer is yes. We even have the following theorem.

THEOREM 2. *Let* $1 \leq p < 2$ *and* diam $D \leq 1$. *Then there is a solution* u *to* (1.1) *such that*

$$\left(\int_D (|u| e^{-\phi})^p\right)^{1/p} \leq C_p \int_D |f| e^{-\phi}$$

provided the right-hand side is finite. Here, C_p *is a constant depending only on* p.

Received 4 February 1991. Revision received 3 September 1991.
Author supported by the Natural Science Research Council.