

PRESCRIBING THE STRAIN OF A DIFFEOMORPHISM AND SOLVABILITY OF THE SINGULAR CAUCHY PROBLEM

GEORGI I. KAMBEROV

CONTENTS

1. Introduction	
1.1. Summary	421
1.2. The eigenvalue problem	422
1.3. Singular theorem of Cauchy-Kovalevski	424
2. Prescribing eigenvalues: principal axes	
2.1. Definitions and notation	426
2.2. Cauchy problem: conditions on the initial data	430
2.3. Proof of Theorem 1.1	435
2.4. Existence of principal axes	437
3. Proof of the singular theorem of Cauchy-Kovalevski	
3.1. Setting the stage	438
3.2. Notation, definitions, basic facts	441
3.3. Main lemmas and their proofs	442
3.4. Proof of Theorem 3.1	444

1. Introduction

1.1. Summary. In this article we study diffeomorphisms between Riemannian manifolds in the case when two of the eigenvalues coincide to order zero on a smooth submanifold of codimension one. To prove the existence of such a diffeomorphism, one must consider a singular nonlinear differential system: the rank of the symbol of the system drops on a smooth submanifold Σ , and every covector based on Σ is characteristic for the system. The techniques from differential systems theory do not apply, because they all rest upon the hypothesis of locally constant rank of the symbol. In this article we prove a local solvability theorem for singular differential systems in the real-analytic category and apply it to the eigenvalue problem. In a forthcoming article we use the same local solvability theorem to solve the Ricci problem for symmetric tensor fields whose rank has a jump on a hypersurface.

It is a pleasure to thank Dennis De Turck for introducing us to these problems and for his constant support. We also want to thank Eugenio Calabi, Jerry Kazdan, and Steven Shatz for their constructive interest in this work.

Received 21 December 1990. Revision received 14 August 1991.