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AMPLE WEIL DIVISORS ON K3 SURFACES WITH DU VAL SINGULARITIES

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0.1. The motivation of this paper comes from the study of $\mathbf{Q}$-Fano varieties. $\mathbf{Q}$-Fanos are one of the classes of (singular) varieties that naturally appear in the Minimal Model Program; see [KMM], [Ko], [Mr], or [W] for the introduction.

0.2. Nonsingular Fano varieties (i.e., varieties with the ample anticanonical class $-K_X$) with Picard number $\rho(X) = 1$ were classified by G. Fano and V. A. Iskovskikh; see [I1], [I2]. Among the first steps in this classification are the following.

(i) Find a smooth surface S in the anticanonical linear system $|-K_X|$ (done in [Sh]). By adjunction formula and Kodaira vanishing, it is a K3 surface.
(ii) Restrict $|-K_X|$ on S. It is an ample linear system. Now use the following to obtain the description of $|-K_X|$.

THEOREM 0.2.1 ([SD]). *Let $|D|$ be an ample complete system on a smooth* K3 *surface. Then $|D|$ is either free or has the single base component C of multiplicity one and $|D| = C + |nE|$ where $|E|$ is an elliptic pencil.*

0.3. For the singular $\mathbf{Q}$-Fanos one can try to use the same approach. The first difference is that, if X has a non-Gorenstein singularity, then locally in a neighborhood of such a point a general element of $|-K_X|$ should have Du Val singularity. The second observation is that $-K_X$, restricted on S, is not a Cartier divisor any more but only a Weil divisor such that its multiple is an ample Cartier divisor.

Therefore, we see that in order to work with the singular case we have to consider a $K3$ surface with Du Val singularities and an ample Weil divisor on it.

0.4. Some results in the direction of 0.2.(i) are contained in [A]. Among them is the following theorem.

THEOREM 0.4.1. *Let X be a $\mathbf{Q}$-Fano of degree $d = (-K_X)^3 \geqslant 4$. Then one of the following is true.*

(i) *A general element $S \in |-K_X|$ has (not worse than) Du Val singularities;*
(ii) *X is birationally equivalent to another $\mathbf{Q}$-Fano X_1 such that a general element $S \in |-K_{X_1}|$ has Du Val singularities;*

0.5. This paper gives some answers for the second half of 0.2. Here we study the

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