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MODIFIED ISOPERIMETRIC CONSTANTS, AND LARGE TIME HEAT DIFFUSION IN RIEMANNIAN MANIFOLDS

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In this paper we solve a problem first raised in [4], the background of which is as follows.

Let M be a noncompact Riemannian manifold of dimension $n \geqslant 2$, with associated Laplace-Beltrami operator Δ acting on functions on M and with attendant minimal positive heat kernel $p(x, y, t)$, where $x, y \in M$, and $t > 0$. Thus, if $M = \mathbb{R}^n$, then p is given by the classical Gauss kernel

$$p(x, y, t) = \mathbf{e}_n(x, y, t) =: (4\pi t)^{-n/2} e^{-|x-y|^2/4t}.$$

Our interest, for general M, is in those aspects of the geometry of M related to the inequalities of the type

$$p(x, y, t) \leqslant \text{const.}_\nu \, t^{-\nu/2}, \qquad \nu > 0 \tag{1}$$

for large $t > 0$.

One of the powerful geometric tools used to obtain such estimates is the apparatus of isoperimetric constants.

Definition. To each $\nu > 1$ and open submanifold Ω with compact closure and smooth boundary, associate the *ν-isoperimetric quotient of* Ω, $\mathfrak{I}_\nu(\Omega)$, defined by

$$\mathfrak{I}_\nu(\Omega) = \frac{A(\partial\Omega)}{V(\Omega)^{(\nu-1)/\nu}}$$

where A denotes $(n-1)$-dimensional Riemannian measure and V denotes n-dimensional Riemannian measure.

The *ν-isoperimetric constant of* M, $I_\nu(M)$, is defined to be the infimum of $\mathfrak{I}_\nu(\Omega)$ over all Ω described above.

The basic result which is the backdrop of all we do here is the following theorem. (See [21], [9], [16], and [2]; also see [24], [10].)

THEOREM 1. *If $I_\nu(M) > 0$, $\nu \geqslant 2$, then* (1) *is valid on all of $M \times M \times (0, +\infty)$. Moreover, given any $\delta > 0$, there exists* $\text{const.}_{\nu,\delta} > 0$ *for which*

$$p(x, y, t) \leqslant \text{const.}_{\nu,\delta} \, t^{-\nu/2} e^{-d^2(x,y)/4(1+\delta)t} \tag{2}$$

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