Vol. 64, No. 3 DUKE MATHEMATICAL JOURNAL © December 1991

COMPOSITION OF SOME SINGULAR FOURIER INTEGRAL OPERATORS AND ESTIMATES FOR RESTRICTED X-RAY TRANSFORMS, II

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0. Introduction. Since the composition of two Fourier integral operators fails, in general, to be a Fourier integral operator, it is of interest to describe their composition and obtain sharp estimates for them. However, to do this in complete generality is probably a hopeless task. The purpose of this paper is to introduce, for some pairs of integers (m, n) with $0 \leqslant m \leqslant n$, a class of canonical relations $C \subset T^*X \times T^*Y$, where Y and X are smooth manifolds of dimensions $N + m$ and $N + n$, respectively, with projections $\pi\colon C \to T^*Y$ and $\rho\colon C \to T^*X$ described using the language of singularity theory such that, if A and B are Fourier integral operators from $\mathscr{E}'(Y)$ to $\mathscr{D}'(X)$ associated with C, then the composition B^*A belongs to a class of pseudodifferential operators with singular symbols [21, 15] on Y, denoted $I^{p,l}(\Delta_{T^*Y}, \Lambda_{\pi(L)})$. From the known results [11] concerning this latter class, one obtains optimal L^2 estimates and can construct relative left parametrices. These canonical relations, which we call *(m, n)-fibered folding canonical relations*, include the canonical graphs ($m = n = 0$), symplectic reductions of codimension n involutive submanifolds ($m = 0$, n arbitrary), and the fibered folding canonical relations of [20, 10, 13] ($m = n = 1$). For general m and n they arise naturally in three related ways: in integral geometry, in the study of oscillatory integrals with degenerate phase functions, and in the construction of symplectic "resolution of singularities".

The singularity classes to which the mappings π and ρ belong, which generalize submersions with folds and blowdowns, respectively, are introduced in §1. In §2 we give motivational examples of symplectic and canonical relations whose projections belong to these classes, from the points of view of a suitable generalization of symplectic polar coordinates and oscillatory integrals. These lead to the general definition of the (m, n)-fibered folding canonical relations. In §3 a very weak normal form is established, which allows the parametrization of an (m, n)-fibered folding canonical relation by a phase function close to the examples of §2. Using an extension of the iterated regularity arguments for the case $m = n = 1$ in [12], we prove the following theorem.

THEOREM 0.1. *For $1 \leqslant m \leqslant n$, let $C \subset (T^*X\backslash 0) \times (T^*Y\backslash 0)$ be a nonradial (m, n)-fibered folding canonical relation, and $A \in I^r(C; X, Y)$ and $B \in I^{r'}(C; X, Y)$ properly supported Fourier integral operators. Then*

$$B^*A \in I^{r+r'+((n-m)/2)-\nu(\pi),\,\nu(\pi)}(\Delta_{T^*Y}, \Lambda_{\pi(L)}) \tag{0.2}$$

Received 25 January 1991. Revision received 4 June 1991.