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BOUNDARY SINGULARITIES OF SOLUTIONS OF SOME NONLINEAR ELLIPTIC EQUATIONS

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1. Introduction. In this article we study some new types of problems related to the boundary behaviour of the solutions of semilinear elliptic equations of the type

$$-\Delta u + g(u) = 0. \tag{1.1}$$

We assume that G is a bounded open subset of $\mathbb{R}^N$ with a C^2 boundary ∂G, that g is a continuous, nondecreasing, real-valued function, and that u is C^2 in G. The three problems we shall study in detail are the following.

(P1). The Dirichlet problem with measure boundary data. This means that we look for a solution u of (1.1) in G such that u be equal to some measure μ on ∂G.

(P2). The removability of any isolated singularity of u lying on ∂G. More specifically, we shall prove that, under suitable growth conditions on g, any u satisfying (1.1), which is continuous in $\overline{G}\backslash F$, where F is a discrete subset of ∂G, and which coincides on $\partial G\backslash F$ with some given function ϕ continuous on ∂G, can be extended to $\overline{G}$ as a continuous solution of (1.1) in some appropriate sense.

(P3). If we assume that a discrete set of ∂G is not a removable singularity for u, can we describe the behaviour of u near the singular set?

When the boundary singularity problem is replaced by an interior singularity problem (that is, the possible singular set lies in G), those three types of problem have been studied by numerous mathematicians. The pioneering work is due to Serrin [23], [24], where Δ is replaced by a very general quasilinear elliptic operator in divergence form and where the perturbation term is dominated. When $g(r)$ behaves asymptotically as a power of r, interior problems corresponding to (P1) and (P2) have been studied by Bénilan and Brézis [5], Brézis and Véron [12], Vazquez and Véron [28], [29], and Baras and Pierre [3]. As for problem (P3), it has been initiated by Véron [31]. A more powerful approach has been given by Chen, Matano, and Véron [14].

The model problem is the following. Let $G = \mathbb{R}^{N-1} \times \mathbb{R}_*^+$, $x = (x', x_N)$ the coordinates in G and $q > 1$; can we find particular solutions of

$$\begin{cases} -\Delta u + |u|^{q-1}u = 0 & \text{in } G, \\ \qquad\qquad\quad u = 0 & \text{on } \partial G\backslash\{(0,0)\} \end{cases} \tag{1.2}$$

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