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BOUNDARY REGULARITY OF A CANONICAL SOLUTION OF THE $\bar{\partial}_b$ PROBLEM

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§1. Introduction.

Statement of results. We shall discuss the L^2-existence and boundary regularity of a canonical solution of the equation $\bar{\partial}_b v = \alpha$ using the formalism of the $\bar{\partial}_b$-Neumann problem for certain smoothly bounded open subdomains of strictly pseudoconvex CR-manifolds whose CR-structure is endowed with a Levi metric. Each subdomain Ω is assumed to have two special properties: the boundary must be everywhere noncharacteristic to the $\bar{\partial}_b$-complex, and the domain must admit a defining function $r: \Omega \to R$ that depends only on the real and imaginary parts of a single CR-function z. The latter restriction is a feature common to most discussions of local solvability of the $\bar{\partial}_b$-problem [13], [6], [15], [16], [18], but the former restriction is more stringent than that imposed by previous investigators. Indeed, the noncharacteristic condition imposes nontrivial global constaints on the topology of the boundary of Ω [20] which imply that Ω cannot be topologically a ball. We use the stronger hypothesis to obtain new results concerning the boundary regularity of the solution. There are two main results to discuss. Theorem 3.10 provides a geometrical condition that is sufficient for the basic L^2-estimate $\|v\|^2_{L^2(\Omega)} \leqslant C\|\bar{\partial}_b v\|^2_{L^2(\Omega)} + \|\bar{\partial}_b^* v\|^2_{L^2(\Omega)}$ to hold for test forms supported near the boundary which satisfy the boundary condition relevant to solvability of the $\bar{\partial}_b$-problem on Ω. The condition is obtained by modifying the calculations of Kuranishi [13, I] using a frame-independent, covariant derivative notation that exposes the intrinsic geometry of the situation. The geometrical condition is a point-wise constraint on the curvature tensor of the Webster connection induced by the Levi metric and on the divergence of a certain vectorfield obtained by covariant differentiation of a unit-length section of the rank-one complex vectorbundle dual to $\bar{\partial}_b \bar{z}$. The vectorfield is the formal analog in the CR-setting of the mean curvature vectorfield associated to the foliation of a Riemannian manifold by real hypersurfaces. An interesting property of this vectorfield is that it is conformally invariant under analytic reparametrization of the coordinate z. Another noteworthy feature of this geometrical condition is that it is determined exclusively by the function z rather than by the defining function $r(z, \bar{z})$.

The second half of the paper is devoted to the topic of global regularity of the weak solution of the equation $(\bar{\partial}_b \bar{\partial}_b^* + \bar{\partial}_b^* \bar{\partial}_b)u = \alpha$ on Ω in a space of forms that satisfy the boundary condition relevant to solvability of the $\bar{\partial}_b$-problem. Theorem 7.5 asserts that, if a basic $L^2(\Omega)$-existence estimate is satisfied by the test forms in

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