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# A PERTURBATION RESULT IN PRESCRIBING SCALAR CURVATURE ON $S^n$

SUN-YUNG A. CHANG AND PAUL C. YANG

**§0. Introduction and statement of theorem.** On $S^n = \{x \in \mathbb{R}^{n+1} \mid |x| = 1\}$ the standard metric $g_0 = ds^2 = \sum_{i=1}^{n+1} dx_i^2$ has constant scalar curvature $R_0 = n(n-1)$. A conformally related metric $g$ is usually written as $g = e^{2w}g_0$ when $n = 2$ and as $g = u^{4/(n-2)}g_0$ when $n > 2$. The scalar curvature function $R$ ($R = 2K$ when $n = 2$) of $g$ is given by the differential equation

$$\Delta w + Ke^{2w} = 1 \tag{0.1}$$

$$C_n \Delta u + Ru^{(n+2)/(n-2)} = R_0 u, \qquad \text{where} \quad C_n = \frac{4(n-1)}{n-2}. \tag{0.1$'$}$$

The problem is to decide which function on $S^n$ can be the scalar curvature of a conformal metric. This question was studied in [A], [B-Co], [Bo], [Bo-E], [C-Y, 1], [C-Y, 2], [Chen-D], [E-S], [Han], [Ho], [K-W], [M, 2], [S-Z], and [Z]. In particular, in [K-W] Kazdan and Warner gave the necessary conditions

$$\int \langle \nabla K, \nabla x_j \rangle e^{2w} = 0, \qquad j = 1, 2, 3, \qquad \text{when} \qquad n = 2 \qquad \text{and} \tag{0.2}$$

$$\int \langle \nabla R, \nabla x_j \rangle u^{2n/(n-2)} = 0, \qquad j = 1, 2, \ldots, n+1 \qquad \text{when} \qquad n > 2. \tag{0.2$'$}$$

A conformal transformation $\varphi$ of $S^n$ acts on the set of conformal metrics $g = u^{4/(n-2)}g_0$ by $\varphi^* g = (T_\varphi u)^{4/(n-2)} g_0 (= e^{2(T_\varphi w)} g_0)$ where

$$T_\varphi w = w \circ \varphi + \frac{1}{2} \log \det |d\varphi|$$

$$T_\varphi u = (u \circ \varphi) \cdot |\det d\varphi|^{(n-2)/2n}.$$

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