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QUASI-PROJECTIVE EMBEDDINGS OF NONCOMPACT COMPLETE KÄHLER MANIFOLDS OF POSITIVE RICCI CURVATURE AND SATISFYING CERTAIN TOPOLOGICAL CONDITIONS

WING-KEUNG TO

§1. Introduction and statements of results. In the study of compactifications of complete Kähler manifolds of positive Ricci curvature, Mok [21] proved the following theorem.

THEOREM (Mok [21, Main Theorem]). *Let X be an n-dimensional complete Kähler manifold of positive Ricci curvature. Suppose for some base point x_0 and some positive constants C and c, we have*

(i) $|\text{Scalar Curvature}| < C/d^2(x_0; x)$,
(ii) $\int_X \text{Ric}^n < \infty$, *and*
(iii) $\text{Volume}(B(x_0, r)) \geqslant cr^{2n}$,

where $d(\ ;\ )$ denotes the geodesic distance function and $B(x_0, r)$ denotes the geodesic ball centered at x_0 and of radius r. Then X is biholomorphic to a quasi-projective variety.

Conditions (i) and (iii) of the above theorem are not automatically satisfied for manifolds satisfying the remaining conditions of the above theorem. In [15] Klembeck constructed a complete Kähler metric on C^n of positive sectional curvature which satisfies condition (ii) and violates conditions (i) and (iii) of the above theorem when $n \geqslant 2$. (See the appendix for a verification of these properties.) This article grows out of an attempt to prove the above theorem without the assumption on the lower bound of the growth of volume of geodesic balls while relaxing the decay conditions of the curvature tensor.

As in Mok-Zhong [26], we say that a topological manifold is of finite-topological type if it is homotopic to a finite CW complex. Our main result is the following theorem.

MAIN THEOREM. *Let X be an n-dimensional noncompact complete Kähler manifold of positive Ricci curvature and of finite topological type. Denote the Kähler form, geodesic distance functions, holomorphic bisectional curvature, the Ricci form, geodesic balls of X by ω, $d(\ ;\ )$,* Bisect(,), Ric *and* $B(\ ;\ )$, *respectively. Suppose for some base point x_0 that there exist positive constants k, k', k'' and a positive real number p such that for $R > 0$*

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