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KILLING HANDLES OF INDEX ONE STABLY, AND π_1^∞

V. POÉNARU

1. Introduction. The main result of this paper is the following theorem.

THEOREM. *Let V^3 be an open, simply-connected 3-manifold. We assume that for some positive integer n the manifold $V^3 \times D^n$, where D^n is the standard smooth n-dimensional ball (with its boundary S^{n-1}), admits a handlebody decomposition having the following two properties.*

(a) *With respect to the standard DIFF product structure of $V^3 \times D^n$, this handlebody decomposition is piecewise smooth.*
(b) *Our handlebody decomposition is without handles of index one.*

Then, for any compact set $K \subset V^3$, there is a compact smooth simply-connected submanifold $U^3 \subset V^3$ such that $K \subset \mathring{U}^3$. In other terms, the existence of a handlebody decomposition without handles of index one for $V^3 \times D^n$ implies that $\pi_1^\infty V^3 = 0$.

Since $V^3 \times D^n$ is both noncompact and also with a very large boundary, namely $V^3 \times S^{n-1}$, it is perhaps worthwhile to explain what we mean here by "handlebody decomposition". One way to think of it runs as follows; we give it here in full detail since we will use again the same notations in Section 5 of this paper. We are given an increasing sequence of compact *bounded* PL-manifolds of dimension $n + 3$

$$X_0^{n+3} \subset X_1^{n+3} \subset \cdots \tag{1.1}$$

such that the following four conditions hold.
(i) $X_0^{n+3} = D^{n+3}_{\text{standard}}$; of course the adjective "standard" used here is relevant only if $n + 3 = 4$.
(ii) One has

$$X_i^{n+3} = X_{i-1}^{n+3} + \{\text{a handle } H_i = D^{\lambda(i)} \times D^{n+3-\lambda(i)}\}, \tag{1.2}$$

and *the index $\lambda(i)$ is always different from one.*
(iii) Since we are here in the PL context where no smoothing of edges or corners is necessary, the intersection $\partial X_i^{n+3} \cap \partial X_{i+1}^{n+3}$ is always exactly the closure of $\partial X_i^{n+3} -$ {the attaching zone of the handle H_{i+1}}. This being said, any given X_i^{n+3} is touched only by *finitely* many handles. In looser terms our handlebody decomposition is "*proper*", or in a more symbolic notation we have $\lim_{n=\infty} H_n = \infty$.
(iv) Clearly, the union $\bigcup_{p=1}^{\infty} X_i^{n+3}$ has a natural structure of a PL-manifold with

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