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HIGHER EQUIVARIANT K-THEORY FOR FINITE GROUP ACTIONS

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Introduction. Let G be a finite group operating on a compact oriented differentiable manifold M. Consider the equivariant K-theory ring of M, which we denote by $K_*(M//G)$. If G is trivial, then $K_*(M) \otimes \mathbb{Q}$ is isomorphic to the cohomology ring $H^*(M, \mathbb{Q})$ of M via the Chern character. In general, however, $K_*(M//G) \otimes \mathbb{Q}$ is not isomorphic to the equivariant cohomology ring $H_G^*(M, \mathbb{Q})$. For example, if M is a point, $K_0(M//G)$ is the ring of representations of G while $H_G^*(M) \otimes \mathbb{Q} = \mathbb{Q}$. Recently, G. Segal proved the following formula. (See [Hirzebruch–Höfer].) Let $\mathscr{R}$ be a set of representatives for the conjugacy classes of G. For each $s \in G$ let us denote by $C(s)$ the centralizer of s in G. The group $C(s)$ acts on the fixed point set M^s. Then there is a canonical isomorphism of graded $\mathbb{C}$-vector spaces

$$K_*(M//G) \otimes \mathbb{C} \cong \prod_{s \in \mathscr{R}} H_{C(s)}^*(M^s, \mathbb{C}).$$

This isomorphism given by Segal is not an isomorphism of rings. However, J. Block and Brylinski independently showed that the rings above are actually naturally isomorphic as $\mathbb{C}$-algebras. Since $H_{C(s)}^*(M^s, \mathbb{C}) \cong H^*(M^s, \mathbb{C})^{C(s)} \cong K_*(M^s)^{C(s)} \otimes \mathbb{C}$, this last result may be stated as the existence of a canonical isomorphism of graded $\mathbb{C}$-algebras

$$(*) \qquad K_*(M//G) \otimes \mathbb{C} \cong \prod_{s \in \mathscr{R}} K_*(M^s)^{C(s)} \otimes \mathbb{C}.$$

The isomorphism above is not defined over $\mathbb{Q}$; for example, if M is a point, then $K_0(M//G)$ is the ring of representations of G, and $K_0(M//G) \otimes \mathbb{Q}$ will not be a product of copies of $\mathbb{Q}$, in general.

Now assume that G is a finite group and k is a field of characteristic p (possibly $p = 0$). Let n be the least common multiple of the orders of all the elements of G of order prime to p. (This means all the elements if $p = 0$.) Suppose that k contains all the n-th roots of 1 and set $\Lambda = \mathbb{Z}[1/|G|]$. Let X be a separated noetherian regular scheme of finite Krull dimension over k carrying an ample line bundle and assume that G acts on X as a scheme over the field k. The purpose of this paper is to give a formula for $K_*(X//G) \otimes \Lambda$, where

$$K_*(X//G) = \bigoplus_{i=0}^{\infty} K_i(X//G)$$

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