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LOCAL SOLVABILITY IN A CLASS OF OVERDETERMINED SYSTEMS OF LINEAR PDE

G. A. MENDOZA AND F. TREVES

CONTENTS

Introduction. The present work extends to vector fields with $\mathscr{C}^\infty$ coefficients the main result about vector fields with analytic coefficients in [T3]. Thus we consider n smooth complex vector fields $L_1, \ldots, L_n$, in an open neighborhood of the origin, U, in $\mathbb{R} \times \mathbb{R}^n$; they are assumed to be linearly independent over the complex numbers. We shall be concerned with the local solvability of the system of differential equations

$$L_j u = f_j, \qquad j = 1, \ldots, n. \tag{1}$$

We reason under the hypothesis of local integrability: there is a $\mathscr{C}^\infty$ solution in U of the homogeneous equations

$$L_j Z = 0, \qquad j = 1, \ldots, n \tag{2}$$

such that $dZ \neq 0$ at every point of U. Possibly after contracting U about the origin, we can select the coordinates, $x, t_1, \ldots, t_n$, in such a way that $Z = x + i\varphi(x, t)$ with $\varphi(0, 0) = 0$. Furthermore, after a substitution of the vector fields L_j by linear combinations of them with $\mathscr{C}^\infty$ coefficients, we may assume, and we shall do so from

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