

LOCAL SOLVABILITY IN A CLASS OF OVERDETERMINED SYSTEMS OF LINEAR PDE

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Introduction. The present work extends to vector fields with \mathcal{C}^∞ coefficients the main result about vector fields with analytic coefficients in [T3]. Thus we consider n smooth complex vector fields L_1, \dots, L_n , in an open neighborhood of the origin, U , in $\mathbb{R} \times \mathbb{R}^n$; they are assumed to be linearly independent over the complex numbers. We shall be concerned with the local solvability of the system of differential equations

$$(1) \quad L_j u = f_j, \quad j = 1, \dots, n.$$

We reason under the hypothesis of local integrability: there is a \mathcal{C}^∞ solution in U of the homogeneous equations

$$(2) \quad L_j Z = 0, \quad j = 1, \dots, n$$

such that $dZ \neq 0$ at every point of U . Possibly after contracting U about the origin, we can select the coordinates, x, t_1, \dots, t_n , in such a way that $Z = x + i\varphi(x, t)$ with $\varphi(0, 0) = 0$. Furthermore, after a substitution of the vector fields L_j by linear combinations of them with \mathcal{C}^∞ coefficients, we may assume, and we shall do so from

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