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GALOIS REPRESENTATIONS ASSOCIATED TO SIEGEL MODULAR FORMS OF LOW WEIGHT

RICHARD TAYLOR

Introduction. This article has two principal aims. The first is to outline a generalisation of Wiles's notion of pseudorepresentation (see [W]) and at the same time simplify it. This gives a simple axiomatic characterisation of which functions from a group to an algebraically closed field of characteristic zero arise as the trace of a true representation. The proof is a fairly straightforward application of results of Procesi on invariant theory (see [P]). (Procesi uses results of Weyl to characterise the ring of invariants of GL_d acting by simultaneous conjugation on a certain number of copies of the ring of $d \times d$ matrices.) We define a pseudorepresentation to be any map from a group to a ring satisfying these axioms. Thus over an algebraically closed field it is equivalent to a true semisimple representation. Over other rings it gives a more general notion than being the trace of a true representation which seems to be easier to handle. For example, if $T: G \to R$ is a pseudorepresentation and $TG \subset R' \subset R$, then $T: G \to R'$ is a pseudorepresentation. This property is not true for traces of true representations.

We believe that this last property makes pseudorepresentations useful for constructing Galois representations for modular forms via congruence arguments. Such arguments have proved useful in treating cases where the direct use of algebraic geometry has not been possible. (See [DS], [W] and [T2].) The idea is to start with a modular form, say f, to which one would like to attach an l-adic representation and find congruences modulo arbitrarily high powers of l to other modular forms in spaces spanned by forms to which one can attach l-adic representations. If the forms congruent to f are themselves eigenforms of the Hecke algebra, it is not difficult then to piece together the representations modulo powers of l to construct the desired l-adic representation. (See [T3].) However, this in general seems to be difficult to achieve. However, using pseudorepresentation, one can show that congruences to forms which are not eigenvalues of the Hecke algebra suffice. Roughly, the argument is as follows. If we have found a congruence modulo l^n between f and a form in a space V_n spanned by eigenforms of the Hecke algebra, then we get a morphism from the integral Hecke algebra $\mathbb{T}_n$ on V_n to $\mathbb{Z}/l^n$ which takes a Hecke operator T to its eigenvalue on f. (We ignore the question of the field of rationality in this paragraph as it is only a technical complication.) By the assumption on V_n there will be a Galois representation to $\mathbb{T}_n \otimes \mathbb{Q}_l$, although probably not to $\mathbb{T}_n \otimes \mathbb{Z}_l$. However, we do get a pseudorepresentation to $\mathbb{T}_n \otimes \mathbb{Z}_l$ which reduces to give one to $\mathbb{Z}/l^n$ with the desired values at Frobenius elements. From these we can piece

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