Vol. 63, No. 1 DUKE MATHEMATICAL JOURNAL © June 1991

# ALMOST HERMITIAN SYMMETRIC MANIFOLDS II DIFFERENTIAL INVARIANTS

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**1. Introduction.** In this paper, which is a sequel to [2], we shall exploit the local twistor connection uniquely associated with an almost Hermitian structure and Lie algebra cohomology to construct linear differential invariants of AHS structures. The main differential geometric result is that all *standard* flat invariant operators admit analogues, with the same leading order symbol, on all curved AHS manifolds. An operator is called *standard* if it occurs in a Bernstein-Gelfand-Gelfand resolution of a finite-dimensional module over a complex semisimple Lie algebra. Exterior differentials and Dirac operators are of this form. The most important example of an AHS structure is a *conformal* structure, and it was mostly the rather complicated but extremely beautiful nature of conformal geometry which provided motivation for the work of this paper and [2].

Algebraically, the arena for study is a category of (**g**, **k**)-modules, where (**g**, **k**) is a Hermitian symmetric pair, studied via the local (complex) differential geometry of the associated Hermitian symmetric space. In some sense the study of almost Hermitian symmetric spaces is a study of a deformation of this category. We shall not go into that here, but it is a useful viewpoint to bear in mind. We are concerned with homomorphisms of Verma modules induced from a parabolic with Levi factor **k**. Our main algebraic result is a differential geometric construction of these given (relative) Kazhdan-Lusztig polynomials (which determine the Lie algebra cohomology of irreducible quotients of Verma modules). In particuar we show that Kostant modules [9] admit resolutions by subcomplexes of Bernstein-Gelfand-Gelfand resolutions and give (yet another) construction of the socle filtrations of Verma modules in the conformal case. Thus the paper is of interest even in the flat setting.

Perhaps the most intriguing flat invariant operators are those arising from nonstandard homomorphisms of (generalized) Verma modules. The conformally invariant Laplacian is an example. It is possible for the methods of this paper to be obstructed for these so that we do not obtain curved analogues. On the other hand the obstructions themselves are invariants (often tensorial) of the AHS structure. It seems from the calculations of C. R. Graham [15] that these are genuine obstructions, i.e., that *no* curved analogue can exist if these obstructions are nonzero. Following an idea of Bailey, Eastwood and Gover [1], we approach this problem in the conformal case by studying the Lie algebra cohomology of a continuous family of Verma modules and are led to conjecture the existence of an infinite family

Received 17 February 1990. Revision received 6 September 1990.