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KÄHLER MANIFOLDS AND 1/4-PINCHING

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1. Introduction. Given a compact manifold M, the existence of a Riemannian metric with "large curvature" imposes severe restrictions on the topology of M. For example, when the sectional curvatures are positive and better than 1/4-pinched, it is well known that, if M is simply connected, M must be a sphere. In [MM], M. J. Micallef and J. D. Moore show that one can arrive to the same conclusion by assuming only that M has positive complex sectional curvature in the direction of isotropic planes.

The concept of complex sectional curvature was introduced by Y. T. Siu for the proof of his strong rigidity theorem for Kähler manifolds [Si]. Later, J. H. Sampson showed, [Sa], how this notion of curvature occurs naturally in the study of harmonic maps from Kähler to Riemannian manifolds. In particular, he proves a strong result in the case when the metric of the target manifold has nonpositive complex sectional curvature. We recall the statement of Sampson's theorem at the beginning of the third section. With this theorem as starting point we investigate the restrictions coming from the existence of a metric with negative complex sectional curvature. We obtain the following.

THEOREM 1.1. *Let M be a compact manifold of dimension greater than 2. If M admits a metric with negative complex sectional curvature at every point, then M cannot admit a Kähler metric.*

As examples of metrics with negative complex sectional curvature, we have those with negative curvature operator. Also, we will show in the next section that an inequality of M. Berger implies that a metric with negative sectional curvature with pointwise pinching strictly greater than 1/4 has negative complex sectional curvature; and so, the above theorem holds for these metrics. With more effort, one can prove a statement that includes the case when the pinching is greater than or equal to 1/4. More precisely, we prove in the last part of the article the following.

THEOREM 1.2. *Let (M, g) be a compact Kähler manifold, and let h be another metric for M. Assume that (M, h) has negative sectional curvature with pointwise pinching at least 1/4.*

Then (M, h) is a complex hyperbolic spaceform $\pm$ biholomorphic to (M, g).

We should point out that this theorem was known for compact quotients of complex hyperbolic 2-space, [V]. Also, M. Gromov [G] has shown that non-

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