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RELATIVE BIRATIONAL AUTOMORPHISMS OF ALGEBRAIC FIBER SPACES

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0. Introductions. Let X be a projective variety over an algebraically closed field of characteristic zero k. One can define the k-scheme $Bir(X)$ whose k-rational points are in one-to-one correspondence with birational automorphisms of X, [H1, §1]. If X is nonuniruled, the basic structure of its birational automorphism group is given by the following theorem.

THEOREM [H2, (2.1)]. *For a nonuniruled variety X, there exists a (nonsingular) projective model X' whose associated scheme $Bir(X')_{red}$ is a group scheme. The identity component $Bir^0(X')$ is an abelian variety.*

If X'' is another projective model of X such that $Bir(X'')_{red}$ is a group scheme, $Bir(X')_{red}$ and $Bir(X'')_{red}$ are isomorphic as group schemes, [H2, (2.2)].

The present paper begins further investigation on the birational automorphism group of X. With the above result in mind, it can be divided into two parts: the study of the continuous part, namely the abelian variety $Bir^0(X')$, and the study of the discrete part, namely the quotient group $Bir(X')_{red}/Bir^0(X')$.

Let us assume moreover that X has Kodaira dimension $\kappa(X) \geqslant 0$. Let $\phi: X \to Y$ be its canonical fibration, (2.1). Note that $\kappa(X) = \dim Y$ and that a general fiber of ϕ is of Kodaira dimension zero. We consider the following two conjectures concerning the continuous and the discrete parts of $Bir(X')_{red}$, respectively.

CONJECTURE (5.3.2) (dimension formula). $\dim_k Bir(X) = q(X) - q(Y)$.

CONJECTURE (5.3.3) (finitely generatedness). *The discrete part $Bir(X')_{red}/Bir^0(X')$ is finitely generated.*

Since ϕ is unique up to birational equivalence, the right side of the formula (5.3.2) is a numerical birational invariant of X. We proved (5.3.2) in [H1, (3.10)] when X is a minimal model whose multicanonical system $|mK_X|$ is base-point free for some m. (5.3.3) was suggested to us by Kollár. Both of the conjectures are known to hold if $\dim X \leq 2$.

The main result of this paper, Theorems (5.6), (5.7) (see them for the precise statements), claims that each of the two conjectures of X is true if the same is true of the generic fiber X_η of ϕ. Note that since X_η is a variety over a function field which is nonclosed, one has to make a generalization of (5.3.3) (see (5.3.4)). The proofs

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