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ANALYTIC TORSION FOR LINE BUNDLES ON RIEMANN SURFACES

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Introduction. Analytic torsion of vector bundles over compact complex manifolds was originally defined by Ray and Singer in [R-S]. In the case of line bundles over Riemann surfaces, analytic torsion is the determinant of the Laplacian when the Laplacian acts on smooth sections of a flat line bundle. In this paper we derive an explicit expression for analytic torsion, viewed as a function on the space of flat line bundles, using Riemann's theta function and other data from the underlying Riemann surface.

This problem has been considered by other authors, see [A1], [F3], and [Sm], for example. Our approach is different from others in the following respect. Throughout we always keep the underlying Riemann surface fixed, never varying the complex structure. In order to obtain our final expression, we carefully study the behavior of analytic torsion as the line bundle approaches the trivial bundle, when viewing the space of flat line bundles as being parameterized by the Jacobian variety. As a result, our expression for analytic torsion is complete with no undetermined constants.

The paper is organized as follows. In Section 1 we present background material and establish notation. In Section 2 we define analytic torsion and obtain a differential equation involving analytic torsion, viewed as a function on the Jacobian variety of the underlying surface. In Section 3 we solve this differential equation, hence explicitly determining the dependence of analytic torsion on the character. Our solution contains a factor independent of the character, which we determine through our analysis in Sections 4 and 5. Finally in Section 6 we present our completed formula for analytic torsion. In Section 7 we use our result to recover the result of Ray and Singer in the case when the Riemann surface is of genus one and has its unit area flat metric. We conclude by discussing how our result compares to the results in [A1], [F3], and [Sm].

In order to state our main result we would need to establish all of our notation. Therefore, we refer the reader to Theorem 6.1 for a statement of our formula for analytic torsion and to Sections 1, 3 and 6 for the necessary notation.

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