Vol. 62, No. 2 DUKE MATHEMATICAL JOURNAL © March 1991

LOCAL SOLVABILITY OF FIRST ORDER LINEAR OPERATORS WITH LIPSCHITZ COEFFICIENTS

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1. Introduction. Consider the differential operator of order one with smooth complex coefficients

$$\frac{\partial}{\partial t} + \sum_{j=1}^{n} a_j(x, t)\frac{\partial}{\partial x_j} + c(x, t)$$

defined in a neighborhood of the origin of $\mathbb{R}^{n+1}$. After a local change of variables, it can be put in the form

$$L = \frac{\partial}{\partial t} + i \sum_{j=1}^{n} b_j(x, t)\frac{\partial}{\partial x_j} + c(x, t) \tag{1.1}$$

with $b_j(x, t)$ smooth and real. In [5] Nirenberg and Treves introduced condition $(\mathscr{P})$, later known to be equivalent to local solvability, and proved that it implies that equation

$$Lu = f$$

can be solved in a neighborhood U of the origin choosing u in the Sobolev space $H^{-1}(U)$ for $f \in L^2(U)$. Later, Treves [7] improved this result showing that u can be taken in $L^2(U)$ and relaxed the regularity hypothesis on the coefficients, requiring that $b_j(x, t)$ be of class $\mathscr{C}^2$ and $c(x, t)$ be measurable and bounded. The present article shows that Lipschitz continuity of $b_j(x, t)$ is enough to obtain L^2 solvability and that the solution u can be defined on a whole strip $|t| < \varepsilon$. More precisely

THEOREM 1.1. *Assume that L given by* (1.1) *is defined on $\mathbb{R}^n_x \times \mathbb{R}_t$ with real Lipschitz coefficients $b_j(x, t)$ on the principal part and $c(x, t)$ complex, measurable and bounded. If L verifies condition $(\mathscr{P})$, there exist $\varepsilon_0 > 0$ and $C > 0$ such that for every positive $\varepsilon < \varepsilon_0$ there exists a continuous linear operator*

$$G_\varepsilon: L^2(\mathbb{R}^n \times (-\varepsilon, \varepsilon)) \to L^2(\mathbb{R}^n \times (-\varepsilon, \varepsilon))$$

with operator norm $\|G_\varepsilon\| < C\varepsilon$ such that

$$LG_\varepsilon f = f, \qquad f \in L^2(\mathbb{R}^n \times (-\varepsilon, \varepsilon))$$

in the sense of distributions.

Received 26 January 1990.
Author partly supported by *CNPq*.