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FAMILIES OF 'FIRST EIGENFUNCTIONS' FOR SEMILINEAR ELLIPTIC EIGENVALUE PROBLEMS

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1. Introduction. In the paper [4] we considered the following problem. Let H be an infinite dimensional Hilbert space, and let $g(u)$ be a continuous Frechet differentiable map $H \to \mathbb{R}$. Let

$$S_t = \{u \in H \mid \|u\|^2 = t\} \tag{1.1}$$

and

$$\gamma(t) = \sup_{u \in S_t} g(u) \tag{1.2}$$

The function (1.2) has an obvious meaning when $\Omega \subset \mathbb{R}^n$ is a bounded open set, $H = H_0^1(\Omega)$,

$$\|u\|^2 = \int_\Omega |\nabla u(x)|^2 \, dx$$

and

$$g(u) = \int_\Omega u(x)^2 \, dx$$

Then the supremum (1.2) is attained for each $t > 0$ on a multiple of the first eigenfunction of

$$-\Delta u = \lambda u, \qquad u \in H_0^1(\Omega).$$

In [4] we used the expression $\gamma(t)$ given by (1.2) to study semilinear eigenvalue problems. The conclusions of [4] can be summarized as follows:

THEOREM 1.1. *Assume that*

(i) *$g(u)$ is weakly continuous in H.*
(ii) *$g(u)$ does not have a local maximum in $H \setminus \{0\}$.*
(iii) *$(g'(u), u)$ is continuous in H.*

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