Vol. 62, No. 2 DUKE MATHEMATICAL JOURNAL © March 1991

STRUCTURE OF BAKER-AKHIEZER MODULES OF PRINCIPALLY POLARIZED ABELIAN VARIETIES, COMMUTING PARTIAL DIFFERENTIAL OPERATORS AND ASSOCIATED INTEGRABLE SYSTEMS

ATSUSHI NAKAYASHIKI

0. Introduction. In this paper we introduce and study what we call a Baker-Akhiezer (BA) module. It is a higher-dimensional generalization of the totality of Baker-Akhiezer functions studied by Krichever [10] [11] and others in the case of algebraic curves. Our main result (Theorem 2.3) is that the BA-module of a g-dimensional principally polarized Abelian variety (ppAv) is a free $\mathscr{D}$-module of rank $g!$ at a generic point. As a corollary, first we can embed affine rings of Abelian varieties to the ring of matrix differential operators. Second, we obtain a system of nonlinear partial differential equations which is a generalization of the KP-hierarchy. The more precise situation is as follows.

For any pair of a nonsingular projective algebraic variety X and its ample subvariety D we construct a sheaf $\mathscr{F}(X, D)$ on the Picard variety of X. It is defined as a Fourier transform of the sheaf $\mathscr{O}_X(*D)$ with the universal line bundle $\mathscr{P}$ as a kernel. (As for the Fourier transform of sheaves, see Mukai [12] and for $\mathscr{P}$ see §1.) It carries naturally determined $\mathscr{D}$-module structure. It depends on the choice of bases of normalized 1-forms and 1-cycles, but we can determine all the $\mathscr{D}$-module structure in our construction (Proposition 1.9 and Corollary 1.11). Moreover, in the case of ppAv's we can choose unique $\mathscr{D}$-module structure by imposing some conditions to differentials of the second kind. (See §2.) We call $\mathscr{F}(X, D)$ with the $\mathscr{D}$-module structure a BA-module of (X, D). For the case of algebraic curves, the stalk of $\mathscr{F}(X, D)$ at a generic point agrees with the totality of BA functions of any order which are studied by Krichever and others.

In the one-dimensional case the BA function has two important features which are mutually related. The first is that it gives a solution to various nonlinear differential equations such as the KdV equation, the KP equation, etc. The second is that it is a common eigenfunction of a commutative ring of ordinary differential operators. Our BA-modules of ppAv's generalize these two features to higher dimension. Consequently, any generic ppAv can be considered as a "spectral variety". The special feature here is the appearance of matrices of partial differential operators. In fact, as a generalization of the second feature above, the vector of a $\mathscr{D}$-free base of a BA-module is a common eigenfunction of commuting matrix partial differential operators (Corollary 2.5). This stems from the fact that the BA-module of a ppAv is a free $\mathscr{D}$-module of rank greater than one. On the other hand, in the

Received 14 May 1990.