

p -DESCENT IN CHARACTERISTIC p

DOUGLAS L. ULMER

This paper is concerned with the problem of calculating the Selmer group for the multiplication by p isogeny on an elliptic curve over a global field of characteristic p . The motivation for studying this problem comes from our earlier work on universal elliptic curves over Igusa curves, which are expected to have points of infinite order. After developing the machinery of p -descents in general in the first half of the paper, we turn to the universal curves, where we are able to express the Selmer group in terms of modular forms (mod p) of low weights. In particular, we produce a subspace of the Selmer group predicted by earlier L -function computations and the Birch and Swinnerton-Dyer conjecture.

The plan of the paper is as follows. In Section One we establish some notation and prove a few easy lemmas on Selmer groups which reduce the problem to certain cohomology calculations. Next, two modular forms A and B on elliptic curves in characteristic p are defined. The Hasse invariant A is interpreted in terms of finite flat group schemes; a similar interpretation of B occurs later in the paper. Section Three applies these results to calculate the Selmer group for the Frobenius and Verschiebung isogenies for an elliptic curve over a global field of characteristic p in terms of invariants of the base field such as its differentials and generalized Jacobians. In order to make the Selmer group calculation for the multiplication by p isogeny, we require rather detailed information on the kernel of p on a supersingular elliptic curve over a finite field, and Section Four contains this analysis, as well as an application to the Selmer group for p on a constant supersingular elliptic curve over a global field. Section Five, which is the main technical computation, contains a calculation of the local Selmer group for p on an elliptic curve over a local field of characteristic p with good, supersingular reduction. In Section Six we tie up some loose ends for the cases $p = 2, 3$ by using Voloch's explicit descent formulae. Section Seven is devoted to explicitly describing the Selmer groups for Frobenius, Verschiebung and p on the universal elliptic curves over Igusa curves studied in [16] in terms of modular forms (mod p). These groups have surprising extra structure, such as a filtration similar to the Hodge filtration on 1-dimensional deRham cohomology, and a symmetric bilinear form on the Selmer group for p . Finally, Section Eight presents some examples of the universal case and treats explicitly the cases excluded from the general theory of Section Seven.

Explicit formulae for doing p descents in characteristic p have been given by K. Kramer [5] for $p = 2$, and by J. F. Voloch [18] for general p . It is a pleasure to

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