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CLOSED 3-DIMENSIONAL HYPERSURFACES WITH CONSTANT MEAN CURVATURE AND CONSTANT SCALAR CURVATURE

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1. Introduction. Let M be a compact 3-dimensional Riemannian manifold with metric g, volume form vol and scalar curvature κ. In this paper we will prove the following theorem:

THEOREM 1. *Suppose a is a smooth symmetric tensor field on M of type* (0, 2) *and A is the tensor field of type* (1, 1) *corresponding to a via g. Suppose in addition that*

(1.1) $\kappa \geqslant 0$
(1.2) *the field* ∇a *of type* (0, 3) *is symmetric*
(1.3) $d(\text{trace } A) = 0$
(1.4) $d(\text{trace } A^2) = 0.$

Then

(1.5) $d(\text{trace } A^3) = 0.$

Theorem 1 has the following consequence. Suppose M is a 3-dimensional closed hypersurface immersed in a space of constant curvature. Suppose in addition that M has constant mean curvature and constant scalar curvature $\kappa \geqslant 0$. Then M is isoparametric.

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The proof follows.

2. Preliminaries. Restricting to a connected component of M and adding to a a constant times g, if necessary, we may assume without loss of generality that

$$\text{trace } A = 0 \tag{2.1}$$

and that for some nonnegative constant T

$$\text{trace } A^2 = 6T^2. \tag{2.2}$$

If $T = 0$, $a = 0$ and (1.5) obviously holds. From now on we will assume $T \neq 0$. Replacing a by a/T we may also assume $T = 1$. We set

$$f = \text{trace } A^3.$$

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