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OPERATORS ASSOCIATED TO FLAT PLANE CURVES: L^p ESTIMATES VIA DILATION METHODS

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1. Introduction. Let $\Gamma\colon \mathbb{R} \to \mathbb{R}^n$ be a continuous curve. To Γ we associate two operators, the maximal operator $\mathscr{M} = \mathscr{M}_\Gamma$ and Hilbert transform $\mathscr{H} = \mathscr{H}_\Gamma$. These are defined for $x \in \mathbb{R}^n$ and appropriate functions $f\colon \mathbb{R}^n \to \mathbb{C}$ as follows:

$$\mathscr{H}f(x) = p.v. \int_{-\infty}^{\infty} f(x - \Gamma(t))t^{-1}\,dt \tag{1.1}$$

(a principal value integral) and

$$\mathscr{M}f(x) = \sup_{s>0} s^{-1} \int_0^s |f(x - \Gamma(t))|\,dt. \tag{1.2}$$

The boundedness of these operators on $L^p(\mathbb{R}^n)$ has been extensively studied by a number of authors. See [StW3] for a survey of results through 1978, and more recently [Wn], [Ne], [NVWWn1], [NVWWn2], [NVWWn3], [Ch1], [Ch2], [Ch3], [CaW], [Ca *et al*], [DRu], [ChSt], [Wa].

In this paper, we consider certain curves in dimension $n = 2$ of the form $\Gamma(t) = (t, \gamma(t))$, γ a convex function for $t \geqslant 0$. We have two principal results:

(a) We give a new condition sufficient for the L^p-boundedness of both operators, $1 < p < \infty$, extending the class of curves for which L^p-boundedness is known. Work done during the 1970s on these problems relied heavily upon dilation groups and homogeneity—for example to estimate oscillatory integrals and to prove the boundedness of certain "classical" singular integrals. One feature of the current paper is the introduction of a family of dilations and corresponding scaling arguments to treat curves which are neither homogeneous nor even approximately so. Another feature, as in [Ch2], is the use of a Calderón-Zygmund theory of "classical" singular integrals in a setting which is not a space of homogeneous type in the sense of Coifman and Weiss [CfWe]. (In §3, see the remark following (3.14).) In fact, in our theory the basic balls centered at the origin can be any nested family of open, balanced (i.e., symmetric), bounded, convex sets shrinking to the origin. See §1.3

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