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DIMENSION OF HOLOMORPHIC EXTENSIONS OF COMPLEX SPACES

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Introduction. A holomorphic extension of a complex space X is a complex space Y together with a holomorphic map $\alpha: X \to Y$ such that the adjoint map $\alpha^*: O(Y) \to O(X)$, $f \mapsto f \circ \alpha$, between the global function algebras is bijective. A maximal holomorphic extension is called an *envelope of holomorphy* (see §3). One reason why envelopes of holomorphy have played an important role in the development of complex analysis is that they usually have nicer properties than the original space, although the global functions have not been altered. Until now, no general relationship between the dimension of the envelope of holomorphy $H(X)$ of a complex space X and the dimension of X itself has been known, not even when X is Stein. The simpler question of whether the envelope of holomorphy $H(X)$ of a complex space X is finite-dimensional if the dimension of X is finite has also not been answered. It would be natural to expect that the dimension of $H(X)$ is at most the same as the dimension of X. One goal of this paper is to prove that this is the case; simple examples show that $\dim H(X) < \dim X$ can occur.

Actually, a more general result is proved here. It is shown that the joint spectral image of every holomorphic map $f: X \to \mathbb{C}^n$ is pluripolar in $\mathbb{C}^n$, when $\dim X$ is smaller than n (see §1). The proof employs the theory of plurisubharmonic functions, more precisely, the solution of the Levi problem in $\mathbb{C}^n$ and a deep theorem of Josefson ([Jos]) which shows that locally pluripolar subjects of $\mathbb{C}^n$ are globally pluripolar. The main consequence of this property about the joint spectral image is that $\dim Y \leqslant \dim X$ holds for any holomorphically spreadable holomorphic extension Y of a complex space X. The same is not true if the extension Y is not holomorphically spreadable (see §2). With this theorem, it is now possible to eliminate the assumption of dimension in several existence theorems on the envelope of holomorphy (see §3).

A further application is the proof of the existence of envelopes of holomorphy in the category C_E of all normal connected complex spaces having at least one holomorphically spreadable holomorphic extension. The proof uses the fact that such a space X also has a normal holomorphically spreadable holomorphic extension. Moreover, if X has an irreducible envelope of holomorphy $H(X)$ in the category of all complex spaces, the envelope of holomorphy of X in C_E is the normalization of $H(X)$.

In this paper, a *complex space* always means a reduced complex space with countable topology.

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