Vol. 58, No. 3 DUKE MATHEMATICAL JOURNAL © June 1989

HYPERBOLIC SURFACES IN $\mathbb{P}^3$

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CONTENTS

0. Introduction. Complex projective varieties which are *hyperbolic* in the sense of Kobayashi [Ko] have attracted recent attention because of their conjectured diophantine properties. For example, Lang [La1, La2] has conjectured (among other things) that any hyperbolic complex projective varity which is defined over a number field K can contain at most finitely many points which are rational over K; this conjecture may be regarded as a higher dimensional analogue of the Mordell conjecture. There are, however, very few known examples of hyperbolic varieties.

The purpose of this paper is to construct smooth hyperbolic surfaces in $\mathbb{P}^3$. Previously the only known examples of such surfaces were the Brody-Green surfaces [BrGr] defined in homogeneous coordinates W, X, Y, Z by

$$W^d + X^d + Y^d + Z^d + \varepsilon(WX)^{d/2} + \varepsilon(YZ)^{d/2} = 0$$

where $d \geqslant 50$ is even and $\varepsilon \neq 0$ is sufficiently small.

We consider smooth surfaces $M \subset \mathbb{P}^3$ of degree d such that each monomial in the degree d homogeneous defining polynomial of M contains one of the homogeneous coordinates raised to the pth power for some $p > (3d + 10)/4$. We show that, for such a surface, the image of any holomorphic map $\mathbb{C} \to M$ is contained in certain curves of genus $\leqslant 1$ and degree $\leqslant d^2$ (see Section 6); in particular, *M is hyperbolic iff it contains no such curves.* We obtain a result for certain higher dimensional hypersurfaces as well (Theorem 6.1).

Received January 18, 1988. Revision received October 10, 1988. Research supported by a Sloan Doctoral Dissertation Fellowship.