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CONSTRUCTION OF P.S.H. FUNCTIONS ON WEAKLY PSEUDOCONVEX DOMAINS

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1. Introduction. Let Ω be a smoothly bounded pseudoconvex domain. An example due to Kohn-Nirenberg [10] shows that even when $\partial\Omega$ is real analytic, a point $p \in \partial\Omega$ does not always have a holomorphic separating function.

When $\Omega \subset\subset \mathbb{C}^2$ is of finite type, Bedford and Fornaess [1] proved the existence of a continuous mapping

$$F: \partial\Omega \times \bar{\Omega} \to \mathbb{C}$$

such that for each $p \in \partial\Omega$ the function $F(p, \cdot) \in A(\bar{\Omega}) = \mathscr{C}(\bar{\Omega}) \cap H(\Omega)$ and is a peak function at p.

In this paper we produce a different proof of this result. Bedford and Fornaess used a blowing up technique, here our approach is to construct entire functions dominated by subharmonic functions, using L^2 methods.

The main object of the paper is the construction of plurisubharmonic peak functions with good estimates. More precisely we obtain the following theorem.

THEOREM A. *Let $\Omega \subset\subset \mathbb{C}^2$ be a pseudoconvex domain with smooth boundary and finite type $2k$. There exists a neighborhood U of $\bar{\Omega}$ and a constant C such that for every $\zeta \in \partial\Omega$ there is a plurisubharmonic function $\varphi_\zeta(z)$ on U verifying*

(i) $|\varphi_\zeta(z) - \varphi_\zeta(z')| \leqslant C|z - z'| \; \forall z, z' \in U$
(ii) $\varphi_\zeta(z) \leqslant -|z - \zeta|^{2k} \; \forall z \in \Omega, \; \varphi_\zeta(\zeta) = 0.$

We give also a similar result for convex domains of finite type in $\mathbb{C}^n$; see Theorem 4.1.

As a consequence, using a result due to D. Catlin, we get a simple proof of sharp subelliptic estimates for the $\bar{\partial}$-Neumann problem when Ω is a domain of finite type in $\mathbb{C}^2$ or a convex domain in $\mathbb{C}^n$. Another approach to this problem is given in Fefferman-Kohn [7].

1. Good holomorphic coordinate systems

PROPOSITION 1.1. *Let $\Omega \subset\subset \mathbb{C}^2$ be a domain with $\mathscr{C}^\infty$ boundary and let k be an integer. Then there exists a $\mathscr{C}^\infty$ map $\Phi: \mathbb{C}^2(z, w) \times \partial\Omega(\zeta) \to \mathbb{C}^2(\tilde{z}, \operatorname{Im} \tilde{w}, \zeta)$ on $\{|\tilde{z}, \operatorname{Im} \tilde{w}| < 1\} \times \partial\Omega$ with the following properties:*

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