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CRITERIA FOR TOPOLOGICAL EQUIVALENCE AND A LÊ-RAMANUJAM THEOREM FOR THREE COMPLEX VARIABLES

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1. Introduction. The singularities of functions $f: (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ with isolated critical point at the origin are classified in a series of papers by Arnold. The classification identifies the singularities of functions which differ by an analytic change of coordinates. The classification stratifies the space of functions by the notion of modality. A singularity is said to be *k-modal* if k is the smallest number such that a sufficiently small neighborhood of the germ of the function at the critical point is covered by finitely many k-parameter families of orbits under the action of the group of diffeomorphisms of $\mathbb{C}^{n+1}$. The local behavior of the function near the isolated singularity is therefore of interest. This local behavior can be described in terms of the topological type of the singularity. That is, it is best described in terms of the (local) embedding of the singular variety in a neighborhood of the singular point. The singularities of two functions are *topologically equivalent* if there is a homeomorphism of neighborhoods of the singular points which maps the one singular variety to the other.

For the case of positive modality, singularities are listed by Arnold in families with one or more parameters. It is conceivable that two singularities which are distinguished by Arnold's classification may actually be topologically equivalent. Also, within a family, one must ask whether or not the topological type of the singularities can vary; one asks whether, locally, the singularities within a parameterized family are the same. The Lê-Ramanujam theorem is an answer to this which requires that one condition be satisfied by the family. We note that the condition is one which is met by the families in Arnold's classification.

THEOREM (Lê-Ramanujam). *Let* $f_t: (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$, $n + 1 > 3$, *be a one-parameter family of functions with isolated singularity at the origin. If the Milnor number is constant throughout the family, then the topological type of each singularity in the family is the same.*

Lê-Ramanujam produces an embedding of the Milnor fiber of one function in the family into the Milnor fiber of another function in the family. The crucial step in the argument is to show that *this particular embedding* of the one Milnor fiber into the other can be extended to a diffeomorphism. It succeeds in establishing this in the case of functions of four or more complex variables by using the h-cobordism

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