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BOUNDARY BEHAVIOR OF THE BERGMAN KERNEL FUNCTION IN $\mathbb{C}^2$

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1. Introduction. The Bergman kernel function for a domain $\Omega \subset \mathbb{C}^n$ is the kernel of the Bergman projection operator, the operator projecting $L^2(\Omega)$ onto its holomorphic subspace. The purpose of this paper is to give precise estimates of the kernel function and its derivatives, in both variables, near the boundary of a smooth bounded domain of finite type in $\mathbb{C}^2$.

The nature of the singularity of the Bergman kernel function tells us much about the holomorphic function theory of the domain in question and has been studied extensively since Bergman's original inquiries (see [3]). The survey article [11] of Diederich gives an excellent account of the progress on this question through the asymptotic expansion of the kernel function of strongly pseudoconvex domains given by Fefferman [13]. Of work published since then, we mention that of Catlin [7] and Diederich, Fornaess, and Herbort [12] on the kernel functions of some weakly pseudoconvex domains restricted to the diagonal, and the papers by Bell [1], [2], and Webster [21], where the kernel function is studied off the diagonal and applications to the boundary behavior of biholomorphic mappings are given.

Recall that the kernel function transforms in a simple manner under biholomorphisms. If $F\colon \Omega_1 \to \Omega_2$ is a biholomorphism between bounded domains $\Omega_i \subset \mathbb{C}^n$ and K_{Ω_i} denotes the respective kernel functions, then

$$K_{\Omega_1}(z, w) = |JF(z)| \cdot K_{\Omega_2}(F(z), F(w)) \cdot |\overline{JF(w)}|,$$

where $|J(\cdot)|$ denotes the determinant of the holomorphic Jacobian matrix. If Ω is a smooth bounded pseudoconvex domain in $\mathbb{C}^2$, $z_0 \in b\Omega$, and z^1 is near z_0, we construct a biholomorphism, Φ, depending on z^1 but with holomorphic Jacobian uniformly nonsingular, and prove the following:

THEOREM 1. *Let z_0 be a point of finite type m in the boundary of a smooth bounded pseudoconvex domain Ω in $\mathbb{C}^2$. For $z^1, z^2 \in \Omega$ near z_0, set $\zeta^i = \Phi(z^i)$, $\tilde{\Omega} = \Phi(\Omega)$, and $z' = \pi(z^1)$, where π is the projection onto $b\Omega$. Then there is a neighborhood U of z_0 such that for all 2-indices α, β and $z^1, z^2 \in U$, there exists a constant $C_{\alpha,\beta}$ such that*

$$|D^{\alpha}_{\zeta^1} D^{\beta}_{\bar\zeta^2} K_{\tilde{\Omega}}(\zeta^1, \zeta^2)| \leqslant C_{\alpha,\beta} \sum_{\ell=2}^{m} A_\ell(z')^{(2+\alpha_1+\beta_1/\ell)}$$
$$\cdot \left(|r(\zeta^1)| + |r(\zeta^2)| + |\zeta_2^1 - \zeta_2^2| + \sum_{\ell=2}^{m} A_\ell(z') |\zeta_1^1 - \zeta_1^2|^\ell \right)^{-2-\alpha_2-\beta_2-(2+\alpha_1+\beta_1/\ell)},$$

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