Vol. 58, No. 1 DUKE MATHEMATICAL JOURNAL © February 1989

BOUNDED IMBEDDINGS OF OPEN KÄHLER MANIFOLDS IN C^N

JOHN S. BLAND

The main purpose of this paper is to intrinsically characterize open complex manifolds which are biholomorphically equivalent to smooth bounded analytic subvarieties in C^N with smooth strongly pseudoconvex boundaries. Such manifolds are characterized in two distinct but equivalent ways: the first is a geometric characterization, and the second is function theoretic in nature. (See Theorems 1 through 4 in Section 1 for a precise statement of the main results.)

The first characterization says that if a complete Kähler manifold M has asymptotically smooth hyperbolic geometry (i.e., the geometry looks roughly like that for the hyperbolic ball), then there exist bounded holomorphic functions on M. If, in addition, M admits a global smooth strongly pseudoconvex function (or M has no compact subvarieties), then M imbeds in C^N (for some $N \geqslant n$) as the interior of a compact complex submanifold with a smooth strongly pseudoconvex boundary.

This characterization is geometrically appealing. First, it says that if the geometry of the manifold is sufficiently well modelled by the geometry of the hyperbolic ball at infinity, then it imbeds as a bounded strongly pseudoconvex submanifold of C^N. (In fact, it can be seen that the amount that the asymptotic geometry deviates from that of the ball is the information contained in the Chern-Moser invariants [6].) Second, it is the natural converse to a fact that was already well known (see, e.g., [10]): that if ϕ is a smooth bounded strongly pseudoconvex defining function for a bounded strongly pseudoconvex submanifold of C^N, then the complete Kähler metric with potential $(-\log - \phi)$ has asymptotically constant negative holomorphic sectional curvature, and in fact its geometry models after the ball (see also [2, 3]). However, in this strength also lies its two greatest weaknesses. First, it is clear that this result cannot be generalized to include all bounded pseudoconvex submanifolds of C^N (the weakly pseudoconvex domains do not possess metrics with asymptotically hyperbolic geometry). Second, it is cumbersome to state carefully what "asymptotically hyperbolic geometry" means. Just as the components for the metric blow up at different rates for holomorphic normal directions than for holomorphic tangential directions, so it is also with all higher covariant derivatives. This description must be made in terms of some family of geodesics which tend to infinity; hence the use of exponential coordinates.

The second characterization models itself after characterizations of Stein manifolds. It says that if there exists a bounded smooth strongly pseudoconvex exhaustion function on M which is smooth and strongly pseudoconvex at infinity (see

Received October 28, 1987. Revision received June 13, 1988.