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MODULI SPACES OF DEGREE d HYPERSURFACES IN $\mathbb{P}_n$

FRANCES KIRWAN

Let $\mathscr{M}(n, d)$ denote the moduli space of stable hypersurfaces of degree d in complex projective space $\mathbb{P}_n$. If $\mathbb{C}[X_0, \dots, X_n]_d$ is the space of all homogeneous polynomials of degree d in the variables $X_0, \dots, X_n$ then $\mathscr{M}(n, d)$ is the quotient by $SL(n+1; \mathbb{C})$ of the open subset of $\mathbb{P}(\mathbb{C}[X_0, \dots, X_n]_d)$ consisting of all points which are stable for the action of $SL(n+1; \mathbb{C})$ (see [M2]). Let $\bar{\mathscr{M}}(n,d)$ be the projective "quotient" variety which geometric invariant theory associates to the action on $\mathbb{P}(\mathbb{C}[X_0, \dots, X_n]_d)$ of $SL(n+1; \mathbb{C})$. Then $\bar{\mathscr{M}}(n, d)$ is a compactification of $\mathscr{M}(n, d)$ and hence also of the moduli space of nonsingular hypersurfaces of degree d in $\mathbb{P}_n$. The first aim of this paper is to show that the Betti numbers of $\mathscr{M}(n, d)$ and the intersection Betti numbers of $\bar{\mathscr{M}}(n,d)$ stabilise as $d \to \infty$ for fixed n; more precisely that

$$H_i(\mathscr{M}(n, d); \mathbb{Q}) \cong IH_i(\bar{\mathscr{M}}(n, d); \mathbb{Q}) \cong H_i(BGL(n+1; \mathbb{C}); \mathbb{Q}) \tag{1.1}$$

when $i < 2h(n, d)$, where IH denotes Goresky-MacPherson intersection homology with respect to the middle perversity and

$$h(n, d) \sim \left(\frac{nd}{n+1}\right)^n \Big/ n! \qquad \text{as} \quad d \to \infty. \tag{1.2}$$

(This is proved in §2: see Theorem 2.2 and Lemma 2.1). Here $BGL(n+1; \mathbb{C})$ is the classifying space of the complex general linear group $GL(n+1; \mathbb{C})$. Its cohomology ring is a polynomial ring on n generators in degrees $2, 4, 6, \dots, 2n$.

dim $H_i(BGL(n+1; \mathbb{C}); \mathbb{Q})$

$i =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	. . .
$n = 1$	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	. . .
2	1	0	1	0	2	0	2	0	3	0	3	0	4	0	4	. . .
3	1	0	1	0	2	0	3	0	4	0	5	0	7	0	8	. . .
4	1	0	1	0	2	0	3	0	5	0	6	0	9	0	11	. . .
5	1	0	1	0	2	0	3	0	5	0	7	0	10	0	13	. . .
⋮																

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