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GENERALIZED BERNOULLI NUMBERS AND CONGRUENCE OF MODULAR FORMS

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Dedicated to Professor Ichiro Satake on his sixtieth birthday

Introduction. In [9], Hida studied universal (ordinary) Hecke algebras and congruence of modular forms and proved that the set of local direct summands of universal Hecke algebras corresponds to the set of maximal families of congruent ordinary forms. In this paper, we shall give a sufficient condition for the existence of families of congruent ordinary forms by using generalized Bernoulli numbers.

More precisely, let $M = \mathbf{Q}(\sqrt{-q})$ be an imaginary quadratic field with a prime discriminant q ($\equiv 3 \bmod 4$), $\chi = (\frac{-}{q})$ be the Legendre symbol, and $p \geqslant 5$ be a prime with $\chi(p) = 1$. Letting $\overline{\mathbf{Q}}_p$ (resp. $\overline{\mathbf{Q}}$) be an algebraic closure of $\mathbf{Q}_p$ (resp. $\mathbf{Q}$), we fix embeddings of $\overline{\mathbf{Q}}$ into $\overline{\mathbf{Q}}_p$ and of $\overline{\mathbf{Q}}$ into $\mathbf{C}$ once and for all. We denote by $\mathfrak{P}$ the prime ideal of the ring $\mathcal{O}$ of all algebraic integers in $\overline{\mathbf{Q}}$ corresponding to the fixed embedding of $\overline{\mathbf{Q}}$ into $\overline{\mathbf{Q}}_p$. For each integer $\nu \geqslant 2$, we define two Eisenstein series by

$$(1)\quad E_{1,\nu}(z) = \sum_{n=1}^{\infty} c_{1,\nu}(n)e(nz), \quad c_{1,\nu}(n) = \sum_{0<d\mid n} \chi(n/d)\omega^{1-\nu}(d)d^{\nu-1},$$

$$(2)\quad E_{2,\nu}(z) = -\frac{B_{\nu,\chi\omega^{1-\nu}}}{2\nu} + \sum_{n=1}^{\infty} c_{2,\nu}(n)e(nz),$$

$$c_{2,\nu}(n) = \sum_{0<d\mid n} \chi(d)\omega^{1-\nu}(d)d^{\nu-1}.$$

Here ω is the Teichmüller character mod p, namely, ω is the Dirichlet character mod p satisfying $\omega(a) \equiv a \bmod \mathfrak{P}$; $B_{\nu,\chi\omega^{1-\nu}}$ is the νth generalized Bernoulli number associated with $\chi\omega^{1-\nu}$; and $e(z) = \exp(2\pi iz)$ ($z \in \mathfrak{H} = \{z \in \mathbf{C} \mid \mathrm{Im}(z) > 0\}$). These are modular forms for a congruence subgroup $\Gamma_0(q)$ or $\Gamma_0(qp)$ of $SL_2(\mathbf{Z})$ with character $\chi\omega^{1-\nu}$ and weight ν according as $\nu \equiv 1 \bmod p-1$ or not. Let λ be a Hecke character of M such that

(3a) $\lambda((\alpha)) = \alpha^{\nu-1}$ for all $\alpha \in M^{\times}$ such that $\alpha \equiv 1 \bmod^{\times} \mathfrak{p}$, and

(3b) $\lambda(\mathfrak{a}) \equiv 1 \bmod \mathfrak{P}$ for all ideals $\mathfrak{a}$ of M prime to $\mathfrak{p}$,

where $\bmod^{\times}$ stands for the multiplicative congruence and $\mathfrak{p}$ is the restriction of $\mathfrak{P}$ to M. Then $f_\lambda(z) = \Sigma_{\mathfrak{a}}\lambda(\mathfrak{a})e(N(\mathfrak{a})z)$ is known to belong to the space of cusp

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