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CONDITIONED BROWNIAN MOTION IN PLANAR DOMAINS

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1. Introduction. This paper studies standard complex Brownian motion started at a point x in a connected Greenian domain of the complex plane $\mathbb{C}$ and either conditioned to exit Γ at a given point y of its Martin boundary or conditioned to hit a point y in Γ before leaving Γ. We use $Z(x, y, \Gamma) = \{Z_t(x, y, \Gamma),\ 0 \leqslant t < \infty\}$ to designate either of these processes. Formally, $Z(x, y, \Gamma)$ is the h process with h respectively $K_\Gamma(\cdot, y)$ and $G_\Gamma(\cdot, y)$, where K_Γ is the Martin kernel function of Γ and G_Γ is the Green function of Γ. These are the two basic h processes in Γ; all h processes in Γ are mixtures of them. An extensive discussion of h processes and their uses in potential theory may be found in Doob [6]. See Durrett [7] for an elementary account as well as a description of some of the ways h processes arise in connection with complex variables and partial differential equations. There is more detail on h processes in the next section of this paper.

If A is a Borel subset of $\mathbb{C}$, the area, closure, diameter, complement, and boundary of A are respectively denoted $\sigma(A)$, $\bar{A}$, diam(A), A^c, and ∂A, and the (minimum) Euclidean distance between A and another Borel set B is written $d(A, B)$. The lifetime of $Z(x, y, \Gamma)$ is denoted $\tau(x, y, \Gamma)$. In [4] Cranston and McConnell answer a question of Chung by proving there is an absolute constant K such that

$$E\tau(x, y, \Gamma) \leqslant K\sigma(\Gamma). \tag{1.1}$$

Here we study the processes $Z(x, y, \Gamma)$ under the restriction that Γ is simply connected. Several of our results are related to (1.1). Throughout this paper Ω stands for a simply connected domain which is not the entire plane, that is, which has a Green function, and we suppress x, y, and Ω in the notation by putting $Z = Z(x, y, \Omega)$ and $\tau = \tau(x, y, \Omega)$. We use c, C, c_p, etc., for positive absolute constants, not necessarily the same at each occurrence.

If Q is a square contained in Ω we call it a Whitney square (for Ω) if diam$(Q) \leqslant d(Q, \Omega^c) \leqslant 4$ diam(Q). See [13] for a proof that Ω is a union of Whitney squares $Q_i, 1 \leqslant i < \infty$, which have disjoint interiors. Such a collection of squares is called a Whitney decomposition of Ω. If Q and R are both Whitney squares we define, after P. Jones [11], $\rho(Q, R) = 0$ if $Q = R$, and if $Q \neq R$, $\rho(Q, R)$ is the smallest integer n such that there exist Whitney squares

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