Vol. 57, No. 1 DUKE MATHEMATICAL JOURNAL © August 1988

ON INTERTWINING OPERATORS FOR $GL_N(F)$, F A NONARCHIMEDEAN LOCAL FIELD

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Since the purpose of this paper is rather technical in nature, it may be of some value to begin with an example, by way of motivation. Let E/F be a finite extension of fields and let V be a finite-dimensional vector space over E. Set $A_E = \mathrm{End}_E(V)$ and $A_F = \mathrm{End}_F(V)$, so that A_E is naturally a subalgebra of A_F, and let $\langle\ ,\ \rangle_F$ be the bilinear form on $A_F \times A_F$ defined by $\langle x, y\rangle_F = \mathrm{tr}(xy)$, where tr denotes the usual matrix trace to F. Then $\langle\ ,\ \rangle_F$ is nondegenerate and so may be used to identify A_F with its dual $\mathrm{Hom}(A_F, F)$, this identification assigning an element x in A_F to the functional φ_x given by $\varphi_x(y) = \langle x, y\rangle_F$ for y in A_F. Now the map $\mathrm{res}_{E/F}$ that restricts a functional φ on A_F to A_E clearly maps $\mathrm{Hom}_F(A_F, F)$ onto $\mathrm{Hom}(A_E, F)$; furthermore, if α is any element of E for which $E = F[\alpha]$, then $\ker(\mathrm{res}_{E/F})$ is identified, under the identification above, with the image of A_F under the map A_α: $A_F \to A_F$ defined by $A_\alpha(X) = \alpha X\alpha^{-1} - X$. Thus, we have an isomorphism of $\mathrm{Hom}_F(A_E, F)$ with $A_F/\mathrm{Im}\, A_\alpha$.

Now, *if E/F is separable*, then the restriction of $\langle\ ,\ \rangle_F$ to A_E remains nondegenerate and, in fact, $A_F = A_E \perp \mathrm{Im}\, A_\alpha$. Thus, we may identify $\mathrm{Hom}_F(A_E, F)$ with A_E and, indeed, we have the commutative diagram

$$\begin{array}{ccc} A_F & \xrightarrow[\cong]{\Phi} & \mathrm{Hom}_F(A_F, F) \\ {\scriptstyle P}\downarrow & & \downarrow{\scriptstyle \mathrm{res}_{E/F}} \\ A_E & \xrightarrow[\cong]{\Phi} & \mathrm{Hom}_F(A_E, F), \end{array}$$

where P is the orthogonal projection onto A_E and Φ is the map $x \mapsto \varphi_x$.

However, if E/F is not separable, then the restriction of $\langle\ ,\ \rangle_F$ to A_E is zero. Thus it is of interest to know whether it is possible to replace the map P in the diagram above. Appropriately phrased, this question becomes, Does there exists an (A_E, A_E)-bimodule map S_α: $A_F \to A_F$ for which the following sequence is exact?

$$A_F \xrightarrow{A_\alpha} A_F \xrightarrow{S_\alpha} A_F \xrightarrow{A_\alpha} A_F. \tag{0.1}$$

The purpose of this paper is to describe such a map and to discuss its arithmetical properties in case F is a nonarchimedean local field. In particular,

Received December 15, 1986. Research of first author supported in part by NSF grant DMS83-01946. Research of second author supported in part by NSF grant DMS84-00892 and an NSF Mathematical Sciences Postdoctoral Fellowship.