

ON A PAPER OF ZARROW

HIROKI SATO

Dedicated to Professor Kôtarô Oikawa on his sixtieth birthday

§0. Introduction. A Schottky group is, as defined below, a Kleinian group generated by Möbius transformations which map Jordan curves onto other Jordan curves. If, in particular, circles can be taken as those Jordan curves, the Schottky group will be called *classical* (see the definition below).

In 1974 Marden [2] studied the spaces of these groups and verified that not every Schottky group is classical. In 1975 Zarrow [5] claimed to have obtained an explicit example of a nonclassical Schottky group.

The present paper is a part of our studies in the relationships of classical and nonclassical Schottky groups. In the background of this paper, there is the study of the classical Schottky spaces in Sato [4]. Our result reveals that the above-mentioned Schottky group constructed by Zarrow turns out to be classical. In fact, in his terminology and notation, $S, S^{-1}T$ is a set of classical generators. So there is no Schottky group given in terms of explicit matrices which is known not to be a classical Schottky group, at least none to our knowledge.

§1. Definitions and preliminaries.

1.1. *Definition.* Let $C_1, C_{g+1}, \dots, C_g, C_{2g}$ be a set of $2g$, $g \geq 1$, mutually disjoint Jordan curves on the Riemann sphere which comprise the boundary of a $2g$ -ply connected region ω . Suppose there are g Möbius transformations A_1, A_2, \dots, A_g which have the property that A_j maps C_j onto C_{g+j} and $A_j(\omega) \cap \omega = \emptyset$, $1 \leq j \leq g$. Then g necessarily loxodromic transformations A_j generate a *marked Schottky group* $G = \langle A_1, A_2, \dots, A_g \rangle$ of genus g with ω as a fundamental region. In particular, if all C_j ($j = 1, 2, \dots, 2g$) are circles, then we call A_1, A_2, \dots, A_g a *set of classical generators*. A *classical Schottky group* is a Schottky group for which there exists some set of classical generators.

We say that two marked groups $G = \langle A_1, \dots, A_g \rangle$ and $\hat{G} = \langle \hat{A}_1, \dots, \hat{A}_g \rangle$ are *equivalent* if there exists a Möbius transformation T such that $\hat{A}_j = TA_jT^{-1}$ for all $j = 1, 2, \dots, g$.

Remark. We note that if A_1, \dots, A_g is a set of classical generators and if $\langle \hat{A}_1, \dots, \hat{A}_g \rangle$ is equivalent to $\langle A_1, \dots, A_g \rangle$, then $\hat{A}_1, \dots, \hat{A}_g$ is a set of classical generators.

1.2. In this paper we restrict ourselves to the case $g = 2$. Let A_1 and A_2 be loxodromic transformations. Let λ_j ($|\lambda_j| > 1$), p_j and q_j ($j = 1, 2$) be the

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