Vol. 57, No. 1 DUKE MATHEMATICAL JOURNAL © August 1988

ON THE PROPAGATION OF POLARIZATION IN CONICAL REFRACTION

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1. Introduction. Conical refraction occurs when light enters a biaxial crystal along one of the optical axes, and causes the splitting of the ray into a cone of rays, see [10]. This is caused by the involutive conical singularities of the characteristic set of Maxwell's equations. The propagation of wave front sets for the corresponding scalar wave equation has been analyzed by Melrose and Uhlmann [11]. In this paper, we study the propagation and distribution of polarization sets for systems of equations having an involutive conical singularity of the characteristic set, see Definition 2.1. These include Maxwell's equations in biaxial crystals. The polarization sets, introduced in [2], indicate which components of the solution are the most singular. Actually, we are going to use $H_{(s)}$ polarization sets, where $H_{(s)}$ is the Sobolev space, see [4].

There are two questions to consider when we have an $H_{(s)}$ polarization condition, i.e., some components are in $H_{(s)}$. First, how this $H_{(s)}$ regularity propagates, and secondly how the $H_{(s)}$ singularities in the other components are distributed. In the case of conical refraction the answers to these questions depend on the polarization. There is an invariant definition of real and complex polarization over the conical set, see Definition 2.9. Outside the conical set the polarization propagates along Hamilton orbits, see [2] and [4]. The set of real polarizations over the conical set is foliated by limits of Hamilton orbits. The polarization condition is introduced as a line bundle in which the $H_{(s)}$ polarization is confined. When this line bundle contains (limit) Hamilton orbits, we can define an invariant curvature of the bundle. We get propagation of polarization along the (limit) Hamilton orbits if the curvature satisfies a real first order equation, by Theorems 6.4 and 7.1. When the polarization is complex, there are well-defined complex Hamilton orbits, by Definition 6.6, with invariantly defined curvature. If the line bundle contains complex Hamilton orbits with curvature satisfying a real first order equation, we get propagation of polarization along the orbits, see Theorems 6.7 and 7.2. But there are also other modes of propagation. If the line bundle is not tangent to a limit or complex Hamilton orbit, we can define coherent Hamilton orbits (see Definition 7.4) along which the polarization propagates, by Theorems 6.9 and 7.5.

In this paper, we shall use the classical pseudo-differential operators Ψ^m_{cl} with symbols in $S^m_{\mathrm{cl}} = S^m_{\mathrm{phg}}$, i.e., asymptotic sums of homogeneous terms. Also we shall use the standard symbol classes $S^m_{\rho,\delta}$, $0 \leqslant \delta \leqslant \rho \leqslant 1$, $\delta < 1$, and sometimes the

Received June 18, 1986. Research supported in part by NSF grant MCS-8202241.