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ALGEBRAIC LINKAGE

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Introduction. The purpose of this paper is to prove several results concerning the theory of algebraic linkage. If R is a local Gorenstein ring, I and J are ideals of R, then we say I and J are *linked* (written $I \sim J$) if there is a regular sequence $x_1, \ldots, x_g \subset I \cap J$ such that $(x_1, \ldots, x_g) : I = J$ and $(x_1, \ldots, x_g) : J = I$. This definition is due to Peskine and Szpiro ([7]). Two ideals I and J are in the same linkage class if there are ideals $I_1, \ldots, I_e$ such that $I \sim I_1 \sim \cdots \sim I_e \sim J$.

Two of our main theorems concern ideals in the linkage class of a complete intersection (called *licci* ideals). If the residue class field of R is infinite and R/I is Cohen–Macaulay, we are able to show that $\{p \in \operatorname{Spec}(R) \mid I_p = R_p \text{ or } I_p \text{ is licci in } R_p\}$ is an open set (Corollary 2.11), and are also able to show that if S is a faithfully flat Gorenstein extension of R, then IS is licci if and only if I is licci (Theorem 2.12). The proofs of these theorems rest on another result (Corollary 2.5) which states that if R is a Gorenstein ring with infinite residue class field and R/I is Cohen–Macaulay, then given any sequence of links, $I \sim J_1 \sim \cdots \sim J_e$, one can construct a new sequence of links in R, $I \sim I_1 \sim \cdots \sim I_e$, such that at each step the type and the first t Betti numbers (for a fixed integer t) of R/I_{2i} are bounded by the minimum of the corresponding numbers for R/J_{2i} *and* R/I. In [3] we had already shown this occurs in some extension $R(X_1, \ldots, X_n)$ of R, and the main point of Corollary 2.5 is to achieve this in R by a process of specialization.

The above-mentioned corollary also implies that a licci Gorenstein ideal I admits a sequence of links $I = I_0 \sim I_1 \sim \cdots \sim I_{2e}$, where I_{2e} is a complete intersection, and in each even step, I_{2i} is Gorenstein (Corollary 2.6). Already this result has strong consequences, as was pointed out in [5].

Another main result in the paper says that a local Cohen–Macaulay ring R which is a factor ring of a regular local ring and which is a complete intersection locally in codimension k is actually smoothable in codimension k, i.e., can be written as the specialization of a local ring S satisfying Serre's condition (R_k) (Theorem 3.10). Moreover, the "smoothing" of S can be chosen to be the localization of a suitable generic link of R. As the main feature in the proof we show that the singular locus and the non-complete-intersection locus of the second generic link of a regular ideal coincide (Proposition 3.2). Combining several of the above-mentioned theorems in this paper with some results from [3], we are also able to complete the proof of the following result: Let R be a regular local ring with infinite residue class field, and let I be a licci Gorenstein R-ideal; then R/I is smoothable in codimension 6 (Corollary 3.12).

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