Vol. 55, No. 4 DUKE MATHEMATICAL JOURNAL © December 1987

ON THE WEAK LIMIT OF RAPIDLY OSCILLATING WAVES

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I. Introduction. Conservative, dispersive waves form, and propagate, as packets of rapidly oscillating wavetrains. The mathematical description of this physical process is rather complete for linear waves, but is still in its infancy for nonlinear waves. One mathematical formulation of this problem [1] is as follows: Given a rapidly oscillating nonlinear wave $U(x, t)$, (i) characterize and (ii) derive evolution equations for the weak limit of U (as the wavelengths of the oscillations vanish).

Near-integrable waves [2–6] provide interesting and rich examples for the study of the propagation of rapid, nonlinear waves. One formal approach [7–9] is to construct an asymptotic representation of the wave $U(x, t)$, from which one explicitly calculates both the weak limit and the evolution equations that it satisfies. In the near-integrable framework [8], this approach yields a local (in space x and time t) representation of the wave in the form

$$U^{\varepsilon}(x,t) = W_N\left(\frac{\theta(x,t)}{\varepsilon}; \kappa(x,t), \omega(x,t)\right) + O(\varepsilon), \tag{I.1}$$

where for each κ, ω the function $W_N(\theta)$ is a real-valued function on the N-torus $\mathsf{T}^N \equiv \mathsf{R}^N/2\pi\mathsf{Z}^N$,

$$W_N(\cdot, \kappa, \omega)\colon \mathsf{T}^N \to \mathsf{R}.$$

The N-vectors θ, κ, ω are real-valued functions of x and t related by

$$\frac{\partial}{\partial x}\theta_i = \kappa_i,$$

$$\frac{\partial}{\partial t}\theta_i = \omega_i, \qquad i = 1, 2, \ldots, N. \tag{I.2}$$

Equations (I.2) imply that

$$\frac{\partial}{\partial t}\kappa_i = \frac{\partial}{\partial x}\omega_i, \qquad i = 1, 2, \ldots, N. \tag{I.3a}$$

Received February 25, 1987.