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ON SHIMURA'S CORRESPONDENCE

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0. Introduction. Let us begin by recalling the L-functions associated to an automorphic form ϕ on $GL(r)$ by Godement and Jacquet. Let F be a global field and let S be a finite set of primes including the archimedean ones. Let $F_S = \prod_{v \in S} F_v$, and let R_S be the ring of elements of F that are integers at all places not in S. Assume for simplicity that S is sufficiently large that the domain R_S is a unique factorization ring.

We embed R_S in F_S along the diagonal. Then $\Gamma = GL(r, R_S)$ is a discrete subgroup of $G = GL(r, F_S)$. Let ϕ be a function on $\Gamma \setminus G$. We may define *Hecke operators* acting on the function ϕ as follows: Let p be a prime of R_S. Since R_S is a principal ideal domain, we may find a generator of p, which we denote by the same letter. Let $\xi = \xi(p, i)$ be the matrix

$$\xi(p, i) = \begin{pmatrix} p & & & & & \\ & \ddots & & & & \\ & & p & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{pmatrix}.$$

Here there are to be i diagonal entries equal to p and $r - i$ equal to 1. The Hecke operator $T_{p,i}$ is defined as follows: Let us decompose the double coset $\Gamma \xi(p, i) \Gamma$ into a finite number of left cosets $\bigcup_\nu \Gamma \gamma_{p,i,\nu}$. Then by definition

$$(T_{p,i}\phi)(g) = \sum_\nu \phi(\gamma_{p,i,\nu} g).$$

The Hecke operators form a commutative family of normal operators and hence may be simultaneously diagonalized. If ϕ is an eigenfunction of the Hecke operators, then we call ϕ an automorphic form.

One assumes that there exist eigenvalues $\lambda_{p,i}$ for each $p \notin S$ such that

$$T_{p,i}\phi = \lambda_{p,i} Np^{i(r-i)/2}\phi.$$

There exists a grössencharakter χ such that the "last" eigenvalue

$$\lambda_{p,r} = \chi(p).$$

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