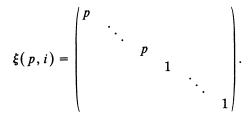
ON SHIMURA'S CORRESPONDENCE

BY DANIEL BUMP AND JEFFREY HOFFSTEIN

0. Introduction. Let us begin by recalling the L-functions associated to an automorphic form ϕ on GL(r) by Godement and Jacquet. Let F be a global field and let S be a finite set of primes including the archimedean ones. Let $F_S = \prod_{v \in S} F_v$, and let R_S be the ring of elements of F that are integers at all places not in S. Assume for simplicity that S is sufficiently large that the domain R_S is a unique factorization ring.

We embed R_s in F_s along the diagonal. Then $\Gamma = GL(r, R_s)$ is a discrete subgroup of $G = GL(r, F_s)$. Let ϕ be a function on $\Gamma \setminus G$. We may define *Hecke* operators acting on the function ϕ as follows: Let p be a prime of R_s . Since R_s is a principal ideal domain, we may find a generator of p, which we denote by the same letter. Let $\xi = \xi(p, i)$ be the matrix



Here there are to be *i* diagonal entries equal to *p* and r - i equal to 1. The Hecke operator $T_{p,i}$ is defined as follows: Let us decompose the double coset $\Gamma\xi(p, i)\Gamma$ into a finite number of left cosets $\bigcup_{\nu}\Gamma\gamma_{p,i,\nu}$. Then by definition

$$(T_{p,i}\phi)(g) = \sum_{\nu} \phi(\gamma_{p,i,\nu}g).$$

The Hecke operators form a commutative family of normal operators and hence may be simultaneously diagonalized. If ϕ is an eigenfunction of the Hecke operators, then we call ϕ an automorphic form.

One assumes that there exist eigenvalues $\lambda_{p,i}$ for each $p \notin S$ such that

$$T_{p,i}\phi = \lambda_{p,i}Np^{i(r-i)/2}\phi.$$

There exists a grössencharakter χ such that the "last" eigenvalue

$$\lambda_{p,r} = \chi(p).$$

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