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THE HODGE STRUCTURES ON THE INTERSECTION HOMOLOGY OF VARIETIES WITH ISOLATED SINGULARITIES

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"Most of the difficulties that one has in understanding mathematics are psychological in nature."

P. Deligne

Introduction. Let X be an n-dimensional variety with isolated singularities Σ. For such spaces, one can write, as is well known, their (middle) intersection homology as

$$IH^i(X) \simeq \begin{cases} H^i(X - \Sigma) & \text{if } i < n, \\ H^i(X) & \text{if } i > n, \\ \operatorname{im}\{H^n(X) \to H^n(X - \Sigma)\} & \text{if } i = n. \end{cases}$$

By [4], the groups on the right-hand side are endowed with mixed Hodge structures; it is known that these mixed Hodge structures are, in fact, pure [8: §3], [11: (1.14)]. Thus, we obtain in this way Hodge structures on $IH^{\cdot}(X)$, as anticipated.* These are clearly the "right" Hodge structures, for one certainly wants

$$H^i(X) \to IH^i(X) \to H^i(X - \Sigma)$$

to be morphisms of mixed Hodge structures. We shall refer to them as the *canonical* Hodge structures on the intersection homology of a variety with isolated singularities. A Hodge complex, in the sense of [4], for these is given in [6].

On the other hand, Hodge structures for intersection homology have been produced, in certain cases, by the "traditional" method, via an isomorphism

$$IH^{\cdot}(X) \simeq H^{\cdot}_{(2)}(X - \Sigma)$$

with L_2 cohomology with respect to a suitable Kähler metric on $X - \Sigma$ [2, 9, 10, 12]. A priori, even their Hodge numbers could be different from those of the canonical Hodge structures. In this article, we prove that these L_2 cohomology

*For a recent general construction, see [16].

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