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ON THE CONSTRUCTION OF INFINITELY MANY CONGRUENCE CLASSES OF IMBEDDED CLOSED MINIMAL HYPERSURFACES IN $S^n(1)$ FOR ALL $n \geqslant 3$

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§1. Introduction. In the study of minimal varieties of general Riemannian manifolds in the setting of geometric measure theory, examples of closed minimal submanifolds in the Euclidean spheres are of basic importance because they actually represent the types of possible isolated singularities in the general setting. In this paper, we shall prove the *infiniteness* of the congruence classes of imbedded, closed minimal hypersurfaces in $S^n(1)$ for all $n \geqslant 3$. We state the results as follows:

THEOREM 1. *For each dimension $n \geqslant 3$, there exist infinitely many, mutually noncongruent examples of imbedded closed minimal hypersurfaces in $S^n(1)$.*

THEOREM 2. *For each dimension $n \geqslant 4$, there exist infinitely many congruence classes of imbedded closed minimal hypersurfaces in $S^n(1)$ which are simply connected.*

The above simple qualitative results are already known for lower dimensions. One has the Lawson's examples [6] in the case of $n = 3$ and the examples of *spherical type* in the case of $n = 4, 5, 6, 7, 8, 10, 12, 14$ in the strongly negative answer to the spherical Bernstein problem [3, 5]. Therefore, the proofs of the above two theorems can be reduced to the construction of infinitely many distinct examples in dimensions $n \geqslant 9$. Actually, we shall prove, instead, the following much more precise theorem:

THEOREM 3. *For each $n \geqslant 4$, there exist infinitely many, mutually non-congruent minimal imbeddings of $S^1 \times S^{n-2}$ (resp. $S^2 \times S^{n-3}$) into $S^n(1)$.*

Technically, the above examples are constructed by the method of equivariant differential geometry. Using the setting of Hsiang-Lawson [4], this amounts to proving the existence of infinitely many global solutions of a specific geometric type of the reduced ODE defined on the orbit space of two specially chosen equivariant systems (cf. §2). The analytical task involved is quite similar to that of [3]. Therefore, in retrospect, the real reason for the existence of such a clean-cut, uniform proof of the above results lies in the *existence* of the two particularly nice equivariant geometric systems on $S^n(1)$. For example, one expects that Theorem 2 should hold for *all simply connected compact* symmetric

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