Vol. 55, No. 1 DUKE MATHEMATICAL JOURNAL © March 1987

# DEFECT RELATIONS FOR HOLOMORPHIC MAPS BETWEEN SPACES OF DIFFERENT DIMENSIONS

YUM-TONG SIU

There is now a rather rich theory of defect relations for holomorphic maps between spaces of equal dimension developed by Nevanlinna, Stoll, Griffiths and many other people. However, for the case of maps between spaces of unequal dimension essentially there is only the theory developed by Weyl–Weyl and Ahlfors when the target space is the complex projective space, and their theory is built on the projective linear structure of the projective space. A partial list of references for these results is given at the end of this paper.

In this paper we derive a defect relation for holomorphic maps between spaces of different dimensions where the target space can be an arbitrary compact complex algebraic manifold. Since the special case of maps from $\mathbf{C}$ to a compact complex manifold of complex dimension two contains already the methods and techniques needed for the general case, in order to avoid unnecessarily complicated notations and to make the arguments more easily understood, we give complete details only for this special case and indicate the easy modifications needed for the general case. Besides the usual techniques in value distribution theory of using the Poincaré-Lelong formula, the calculus lemma of estimating the integrand by its integral, and the concavity of logarithm, a new key point in our argument is to introduce a meromorphic connection so that the "second fundamental form" of a divisor with respect to the connection is zero. More precisely, for the case of a meromorphic map $f$ from $\mathbf{C}$ to a compact Kähler surface $M$, to consider the defect for a nonsingular zero-set of a holomorphic section $s$ of a positive holomorphic line bundle $L$ we introduce a meromorphic connection $\Gamma^{\gamma}_{\alpha\beta}$ for $M$ with the following two properties: (i) For some Hermitian holomorphic line bundle $F$ over $M$ and some holomorphic section $t \not\equiv 0$ of $F$, $t\Gamma^{\gamma}_{\alpha\beta}$ is holomorphic. (ii) Locally on $M$ for any fixed $\alpha, \beta$, $t\mathbf{D}_{\alpha}\partial_{\beta}s$ is a linear combination of $\partial_{\alpha}s$, $\partial_{\beta}s$ and $s$ with smooth coefficients, where $\mathbf{D}_{\alpha}$ is the covariant derivative with respect to $\Gamma^{\gamma}_{\alpha\beta}$. If $f$ satisfies a nondegeneracy condition, then the defect does not exceed any nonnegative number $\alpha$ such that $\alpha$ times the curvature of $L$ dominates the curvature of the tensor product of $F$ and the anticanonical line bundle of $M$. There is also a result for the sum of defects for a family of holomorphic sections of $L$ with nonsingular zero-set if each holomorphic section in the family satisfies condition (ii) above. Very roughly, the reason for the vanishing of the "second fundamental form" is to enable us to use

Received August 4, 1986. Research partially supported by a grant from the National Science Foundation and a Guggenheim Fellowship.