Vol. 55, No. 1 DUKE MATHEMATICAL JOURNAL © March 1987

WEAK (1, 1) BOUNDS FOR OSCILLATORY SINGULAR INTEGRALS

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1. Introduction. Consider the operator $Mf(x) = pv\int_{\mathbb{R}} f(x - t)\exp(it^2)t^{-1}\,dt$ on $\mathbb{R}$. M is known to be bounded on $L^p(\mathbb{R})$ for all $p \in (1, \infty)$, and to map L^∞ to BMO. However the oscillatory factor $\exp(it^2)$ prevents one from applying the standard Calderón-Zygmund method to prove weak type (1, 1) bounds. In this paper we apply a variant of that method to establish weak (1, 1) bounds for a class of operators of which M is the simplest example. A portion of our technique is fairly general and may prove useful in other problems.

THEOREM. *For any polynomial P: $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ and any Calderón-Zygmund kernal K, the operator $Tf(x) = pv\int e^{iP(x,y)}K(x - y)f(y)\,dy$ is of weak type on L^1, with a bound depending only on $\|K\|_{CZ}$ and the degree of P.*

By a Calderón-Zygmund kernel we mean a function K which is C^1 away from the origin, has mean value zero on each sphere centered at the origin and satisfies

$$|K(x)| \leqslant B|x|^{-N} \quad \text{and} \quad |\nabla K(x)| \leqslant B|x|^{-n-1}. \tag{1.1}$$

$\|K\|_{CZ}$ is defined to be the least value of B for which (1.1) holds. Ricci and Stein [RS1, 2] have proved that T is bounded on L^p, $1 < p < \infty$. It is also known that convolution operators with kernels of the form $\exp(i|x|^a)K(x)$, with K Calderón-Zygmund, are of weak type (1, 1) for all $0 < a \neq 1$ [CKS 1, 2].

This result is related to an open question concerning Hilbert transforms along curves. Consider the operator $Hf(x) = pv\int_{\mathbb{R}} f(x_1 - t, x_2 - t^2)t^{-1}\,dt$ in $\mathbb{R}^2$. The question is whether H is of weak type on L^1. If it were, it would follow that M is also; for the proof consider functions of the form $f(x) = g(x_1)e^{ix_2}\chi_{|x_2| \leqslant B}$ and let $B \to \infty$. The techniques employed in this paper have led to a partial result in this direction; see [C3].

It was precisely in the analysis of operators of that type that the class of oscillatory singular integrals treated here arose; see [PS], [RS2] and the references listed there. In particular Phong and Stein [PS] used L^2 bounds for oscillatory integrals as one part of a machine which established the L^2 boundedness of operators associated to Calderón-Zygmund kernels on families of hypersurfaces satisfying certain geometric hypotheses. Motivated by their work, the second of

Received March 3, 1986. Research supported in part by grants from the National Science Foundation.