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WEIGHTED INEQUALITIES FOR THE DYADIC SQUARE FUNCTION WITHOUT DYADIC A_∞

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This paper is dedicated to my parents

§1. Introduction. For $k = 0, \pm 1, \pm 2, \ldots,$ we say that $Q \subset \mathbb{R}^d$ is a dyadic cube of sidelength 2^{-k} if there are integers $j_1, j_2, \ldots, j_d$ so that

$$Q = \left(\frac{j_1}{2^k}, \frac{j_1+1}{2^k}\right) \times \left(\frac{j_2}{2^k}, \frac{j_2+1}{2^k}\right) \times \cdots \times \left(\frac{j_d}{2^k}, \frac{j_d+1}{2^k}\right).$$

If Q is such a cube, we write $l(Q) = 2^{-k}$.

Let $|Q|$ denote Q's Lebesgue measure. For $f \in C_0^\infty(R^d)$ we define:

$$f_Q = \frac{1}{|Q|}\int_Q f$$

$$f_k = \sum_{\substack{Q \text{ dyadic} \\ l(Q)=2^{-k}}} f_Q \chi_Q$$

$$f^*(x) = \sup_k |f_k|.$$

For Q dyadic, $l(Q) = 2^{-k}$, we define:

$$a_Q(f) = (f_{k+1} - f_k)\chi_Q.$$

The functions $a_Q(f)$ are pairwise orthogonal, and $\Sigma_Q a_Q(f)$ converges to f almost everywhere and in $L^2(\mathbb{R}^d)$. It is also easy to see that

$$f_k = \sum_{l(Q) > 2^{-k}} a_Q(f).$$

The dyadic square function $S(f)$ is defined by:

$$S^2(f) = \sum_{x \in Q} \frac{\|a_Q(f)\|_2^2}{|Q|}.$$

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