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p-ADIC K-THEORY OF ELLIPTIC CURVES

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A Yu. I. Manin, avec admiration et sympathie.

Introduction. Let E be an elliptic curve over a number field F. A well-known conjecture of Birch and Swinnerton-Dyer asserts that the rank of the Mordell-Weil group $E(F)$ of rational points of E is equal to the order of vanishing of its L function $L(E; s)$ at $s = 1$. Bloch [2] and Beilinson [1] have proposed that the values of $L(E; s)$ at other integral points are related to other invariants of E, namely its higher K-groups $K_m(E)$, $m \in \mathbb{N}$ (notice that $K_0(E) = \mathbb{Z}^2 \oplus E(F)$). For instance, if we assume for simplicity that E has potentially good reduction, for any integer $i \geqslant 2$, the rank of $K_{2i-2}(E)$ should be equal to the degree $[F:\mathbb{Q}]$ of F over $\mathbb{Q}$. On the other hand, the order of vanishing of $L(E; s)$ at $s = 2 - i$ should be $[F:\mathbb{Q}]$ [14]. Furthermore the leading coefficient of $L(E; s)$ at $s = 2 - i$ is expected to be equal to a regulator defined using $K_{2i-2}(E)$, up to a rational number [1].

The descent theory (or Iwasawa theory) of elliptic curves with complex multiplication gives deep results about the conjecture of Birch and Swinnerton-Dyer [4], and provides it with p-adic analogs. In this paper we investigate how its methods and results can also be used, in some cases, to give p-adic analogs of Bloch and Beilinson's results.

Let us fix an odd prime p and consider the K-theory groups of E with coefficients in $\mathbb{Z}/p^n$, denoted $K_m(E; \mathbb{Z}/p^n)$. We shall be interested in the groups $K_m(E; \mathbb{Z}_p) = \lim_{\leftarrow n} K_m(E; \mathbb{Z}/p^n)$ and $K_m(E; \mathbb{Q}_p/\mathbb{Z}_p) = \lim_{\rightarrow n} K_m(E; \mathbb{Z}/p^n)$.

In Theorems 3.3.2 and 3.4, we use results of Yager [25] and Gross [7] to give examples of curves E and primes p such that the rank of $K_2(E, \mathbb{Q}_p/\mathbb{Z}_p)$ is equal to $[F:\mathbb{Q}]$. The idea of the proof is first to compare $K_2(E, \mathbb{Q}_p/\mathbb{Z}_p)$ with the étale cohomology group $H^2(E, \mathbb{Q}_p/\mathbb{Z}_p(2))$. This can be done using work of Merkurjev-Suslin [11] and Dwyer-Friedlander [5] (Proposition 3.2.). We then compute $H^2(E, \mathbb{Q}_p/\mathbb{Z}_p(2))$ using descent theory and some assumptions of regularity on the prime p.

In paragraph 4, given an elliptic curve with complex multiplication by an imaginary quadratic field K (and defined over K) we define, in some cases, a higher p-adic regulator map

$$r_i: K_{2i-2}(E, \mathbb{Z}_p) \to \mathbb{Z}_p^2$$

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