Vol. 53, No. 4 DUKE MATHEMATICAL JOURNAL © December 1986

CORRECTION TO "ON THE DIFFERENTIABILITY ..." (Duke Mathematical Journal, 52 (1985) 475–484)

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Lemma 3 on page 480 was stated incorrectly, it should read:

LEMMA 3. *$\wedge : S^{n\times n} \to \mathsf{R}$ is Lipschitz continuous. More specifically, for symmetric A and any matrix B with $\wedge(B)$ appropriately ordered (e.g., by analytic continuation from $\wedge(A)$ if $\varepsilon(A) \neq 0$)*

$$|\wedge(A) - \wedge(B)|_\infty \le 2n\|A - B\|$$

where $|\delta|_\infty = \sup_i|\delta_i|$ and $\|C\|$ is the operator norm associated with the standard inner product on R^n.

Proof. We first show that every component of $\wedge(B)$ is within distance $\|A - B\|$ from some component of $\wedge(A)$. Indeed, if $|z - \wedge_i(A)| > \|A - B\|$ $\forall i$, then $\|(z - A)^{-1}\| < \|A - B\|^{-1}$ because A is symmetric. Hence

$$z - B = (z - A)^{-1}\big(I + (z - A)^{-1}(A - B)\big)$$

is invertible and $z \notin \wedge(B)$.

Next, we group the eigenvalues of A into sets G_i as follows

$$G_i = \left\{\lambda_{k_i+1}, \lambda_{k_i+2}, \ldots, \lambda_{k_{i+1}}\right\}$$

where

$$|\lambda_{k_i+1} - \lambda_{k_i}| > 2\|A - B\| \quad \forall i$$

and

$$|\lambda_{k_i+j} - \lambda_{k_i+j+1}| \le 2\|A - B\| \quad \forall k_i + j < k_{i+1}.$$

Let $\delta > 0$ and define

$$\mathscr{M}_i = \left\{z \in \mathsf{C} \,|\, \mathrm{dist}(z, G_i) < (1 + \delta)\|A - B\|\right\}$$

and consider $\wedge(A + \zeta(B - A))$ for $\zeta \in \mathsf{C} + |\zeta| \le 1$. For $|\zeta|$ small there are

Received February 4, 1986.