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THE CAUCHY PROBLEM FOR SEMI LINEAR HYPERBOLIC SYSTEMS WITH DISCONTINUOUS DATA

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1. Introduction. In this paper we will prove the short time existence of solutions to the strictly hyperbolic Cauchy problem:

$$Lu \equiv \partial_t u + \sum_{l=1}^{n} A_l(t,x)\, \partial_{x_l} u = F(t,x,u(t,x)), \tag{1.1}$$

$$u|_{t=0} = g \tag{1.2}$$

where g is piecewise smooth with jumps only over a hypersurface $\Gamma \subset \mathbb{R}^n$.

Indeed, many things have been done recently in the study of weak singularities for smooth enough solutions to semi, or even quasi-linear systems: creation and propagation of singularities, propagation and interaction of progressing waves (J. M. Bony [6], [7], [8], [9], M. Beals [2], [3], R. Melrose-N. Ritter [14], S. Alinhac [1] ...). Also recall that the short time existence of smooth enough solutions is warranted by T. Kato [11] (see also A. Majda [11]). On the other hand, there is of course a great interest in the study of strong singularities. In this direction let us mention A. Majda's work [12] about shocks. (See also G. Métivier [15].) For semi linear systems (1.1) strong singularities means discontinuities of the solutions, and, to begin with, jumps over hypersurfaces. In [17], J. Rauch and M. Reed have begun the study of piecewise smooth solutions, and, in particular, they have solved the Cauchy problem for a two by two system (1.1) with a Cauchy data (1.2) g as indicated above. Unfortunately their method does not seem to apply to a general system, and the aim of this paper is to propose another approach, mainly based on the technics introduced by J. M. Bony, which will allow us to solve (1.1), (1.2) regardless of the size of the system.

In fact, we will solve (1.1), (1.2) by proving the convergence of the obvious iteration scheme (see e.g., A. Majda [11])

$$\begin{cases} Lu_{\nu+1} = F(t,x,u_\nu), \\ u_{\nu+1}|_{t=0} = g. \end{cases} \tag{1.3}$$

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