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STRUCTURE OF POSITIVE SOLUTIONS TO $(-\Delta + V)u = 0$ IN R^n

MINORU MURATA

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§1. Introduction. Positive solutions of second order elliptic equations have fascinated many mathematicians (see [1–5, 7, 9, 11–13, 16, 19–21]). In this paper we investigate the structure of all positive solutions of a stationary Schrödinger equation

$$(1.1) \qquad (-\Delta + V)u = 0 \quad \text{in } R^n,$$

where the potential V is assumed to be a real-valued function belonging to $L_{p,\text{loc}}(R^n)$ with $p > n/2$ for $n \geqq 2$ and $p = 1$ for $n = 1$. We are interested in an explicit representation formula for positive solutions, their asymptotics at infinity, and the Dirichlet problem with a boundary condition at infinity. We are also interested in the stability and unstability of the structure of positive solutions: how the structure is influenced by the change of potentials.

These problems arise from [13–15], where the asymptotic behavior as $t \to \infty$ of a Schrödinger semigroup with a very short range potential and the existence and uniqueness of solutions for elliptic operators like $(-\Delta)^m$ on R^n were investigated. A first step toward extension of the results given there to a Schrödinger operator with a more general potential seems to be establishing an explicit representation formula for all positive solutions. As for the close connection between the positive solutions of a Schrödinger equation and the asymptotic behavior as $t \to \infty$ of the corresponding Schrödinger semigroup, see [13, 18, 19] and references therein.

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