Vol. 53, No. 3 DUKE MATHEMATICAL JOURNAL © September 1986

NONLOCALLY SMOOTHABLE TOPOLOGICAL SYMMETRIES OF FOUR-MANIFOLDS

SLAWOMIR KWASIK AND PIERRE VOGEL

The class of locally smoothable actions was introduced by G. Bredon in [1] as an equivariant analog, in the theory of G-spaces, of the category TOP of topological manifolds. These actions should be regarded as lying somewhere between topological actions and smooth actions. Locally smoothable G-manifolds provide an extremely effective framework for studying the topological properties of smooth G-manifolds. Subsequent research has shown them to be useful and interesting in their own right. The reader is referred to the survey paper of R. Schultz [10].

Although much is known about locally smoothable G-manifolds the picture is quite far from being complete. The purpose of this paper is to shed some light on the difference between the topological and locally smoothable situations. More specifically, we shall show that there exist infinitely many closed manifolds which do not admit locally smoothable involutions but do admit topological ones. As far as we know these are the first such examples of this phenomenon. Our results presented here depend on our previous work [7].

To be more precise we prove the following result:

THEOREM. *There exist infinitely many closed, simply-connected four-dimensional manifolds which do not admit locally smoothable involutions but which do admit topological ones.*

Proof. Let $\phi\colon Z^m \times Z^m \to Z$ be any unimodular, symmetric, even, bilinear form with signature an odd multiple of 8. Let H be a matrix of ϕ. It was proved in [13] that for any integer p there exists a matrix $P_m(p) \in GL_m(Z)$ such that

$$(*) \qquad P_m^{\pm} \equiv {}^tP_m(p) H P_m(p) \mod p,$$

where $P_m^{\pm} = (p_{ij})$ is a $m \times m$ matrix given by

1. $p_{ij} = 0$ if $|i-j| > 1$
2. $p_{ij} = 1$ if $|i-j| = 1$
3. $p_{ii} = 2,\ i = 1, 2, \ldots, m/2$
4. $p_{ii} = 0,\ i = m/2 + 1$
5. $p_{ii} = -2,\ i = m/2 + 2, \ldots, m$

The equation $(*)$ gives a recipe for an equivariant plumbing which yields (see [13]) a four-dimensional simply-connected Z_p-manifold M with boundary $\partial M :=$

Received January 24, 1986.